ACSC/STAT 4720, Life Contingencies II
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Sample Final Examination

This Sample examination has more questions than the actual final, in order to cover a wider range of questions. Estimated times are provided after each question to help your preparation.

1. An individual aged 42 has a current salary of \$76,000 for the coming year. The salary scale is $s_y = 1.05^y$. Estimate the individual's final average salary (average of last 3 years working) assuming the individual retires at exact age 65.

2. An employer sets up a DC pension plan for its employees. The target replacement ratio is 60% of final average salary for an employee who enters the plan at exact age 30, with the following assumptions:

   - At age 65, the employee will purchase a continuous life annuity, plus a continuous reversionary annuity for the employee's spouse, valued at 60% of the life annuity.
   - At age 65, the employee is married to someone aged 63.
   - The salary scale is $s_y = 1.04^y$.
   - Mortalities are independent and given by $\mu_x = 0.0000016(1.092)^x$. The value of the life annuity is based on $\delta = 0.045$. This gives $\overline{a}_{65} = 19.63036$, $\overline{a}_{63} = 19.83656$ and $\overline{a}_{65,63} = 18.7867$.
   - A fixed percentage of salary is payable annually in arrear.
   - Contributions earn an annual rate of 7%.

   Calculate the percentage of salary payable annually to achieve the target replacement rate under these assumptions.

3. The salary scale is given in the following table:

y	s_y	y	s_y	y	s_y	y	s_y
30	1.000000	39	1.350398	48	1.845766	57	2.553877
31	1.033333	40	1.397268	49	1.912422	58	2.649694
32	1.067933	41	1.445983	50	1.981785	59	2.749515
33	1.103853	42	1.496620	51	2.053975	60	2.853522
34	1.141149	43	1.549263	52	2.129115	61	2.961903
35	1.179879	44	1.604000	53	2.207337	62	3.074855
36	1.220103	45	1.660921	54	2.288777	63	3.192585
37	1.261887	46	1.720122	55	2.373580	64	3.315310
38	1.305295	47	1.781702	56	2.461894	65	3.443256

   An employee aged 42 and 4 months has 12 years of service, and a current salary of \$106,000 (for the coming year). She has a defined benefit pension plan with $\alpha = 0.02$ and S_{Fin} is the average of her last 3 years' salary. The employee's mortality is given by $\mu_x = 0.00000195(1.102)^x$. The pension benefit is payable monthly in advance. The interest rate is $i = 0.05$. This results in $\ddot{a}_{65}^{(12)} = 17.15373$ and ${}_{22.66666667}p_{42.33333333} = 0.9901951$. There is no death benefit, and there are no exits other than death or retirement at age 65.

(a) Calculate the EPV of the accrued benefit using the projected unit method under the assumption that the employee retires at age 65. [Calculate the salary scale at non-integer ages by linear interpolation.]

(b) Calculate the employer's contribution for this employee for the year. $[{}_{21.66666667}p_{43.33333333} = 0.9903189.]$

4. The service table is given below:

x	l_x	1	2	3
40	10000.00	118.76	0	0.51
41	9880.73	112.29	0	0.58
42	9767.86	107.16	0	0.65
43	9660.05	101.84	0	0.73
44	9557.49	96.80	0	0.82
45	9459.86	92.02	0	0.93
46	9366.91	87.50	0	1.04
47	9278.37	83.19	0	1.18
48	9193.99	80.11	0	1.32
49	9112.57	75.21	0	1.49
50	9035.87	71.48	0	1.68
51	8962.71	67.92	0	1.89
52	8892.90	64.51	0	2.12
53	8826.26	61.23	0	2.39
54	8762.64	58.07	0	2.69
55	8701.88	55.03	0	3.03
56	8643.83	52.06	0	3.41
57	8588.36	49.18	0	3.84
58	8535.34	46.37	0	4.32
59	8484.64	43.62	0	4.86
60^-	8484.64		1098.84	
60	7385.80	21.70	819.91	5.79
61	6538.40	18.30	611.98	6.38
62	5901.74	10.81	384.29	5.86
63	5500.78	9.14	639.20	6.15
64	4846.29	7.73	351.32	6.10
65^-	4481.14		4481.14	

The salary scale is $s_y = 1.05^y$. The accrual rate is 0.02. The benefit for employees who withdraw is a deferred annual pension with COLA 2%, starting from age 65. For an individual aged 65, we have $\ddot{a}_{65} = 12.85$. The lifetable for an individual who has withdrawn is

x	l_x	d_x
57	10000.00	7.54
58	9992.46	8.22
59	9984.24	8.95
60	9975.29	9.76
61	9965.52	10.65
62	9954.87	11.63
63	9943.25	12.69
64	9930.55	13.86
65	9916.69	15.15

Calculate the EPV of deferred pension benefits made to an individual aged exactly 57, with 16 years of service, whose salary for the past year was \$121,000.

5. A life aged 52 has mortality given in the table below. The yield rate is in another table below

x	l_x	d_x
52	10000.00	9.66
53	9990.34	10.25
54	9980.09	10.88
55	9969.22	11.57
56	9957.65	12.31
57	9945.33	13.12

Term (years)	Yield rate
1	0.038
2	0.041
3	0.043
4	0.039
5	0.038

Calculate the net annual premium for a 5-year term insurance with benefit \$450,000 sold to this life.

6. An insurance company sells 300 one-year life insurance policies to lives aged 48. The death benefit is \$530,000, payable at the end of the year to lives which die during the year. The company uses $q_{48} = 0.00015$ and $i = 0.05$ to calculate the premium for the policy. This results in a net premium of $530000 \times 0.00015(1.05)^{-1} = \$75.71$.

   However, q_{48} is an estimated probability based on past data, and the true value is normally distributed with mean 0.00015 and standard deviation 0.00002. The interest rate cannot be fixed, and the actual interest rate obtained is normally distributed with mean 0.05 and standard deviation 0.003.

   Calculate the expected aggregate profit (at end of year) of the policies, and the variance of this aggregate profit.

7. An insurance company has a 5-year term insurance policy with a death benefit of \$600,000, sold to a life age 57. Mortality is given in the lifetable below.

x	l_x	d_x
57	10000.00	11.19
58	9988.81	11.86
59	9976.95	12.59
60	9964.36	13.38
61	9950.97	14.24
62	9936.73	15.18

   Interest rates for each year are $i = 0.06$ in the first year, and in each subsequent year follow a log-normal distribution with $\mu = -2.8$ and $\sigma = 0.4$.

   (a) The company simulates the following standard normal random variables.

```
-0.3357003 0.5909843 -1.7381753 -1.1146107 -0.1042812 -0.5100134 -0.1351612 -0.8412991
0.9334425 0.7837861
```

   Using as many of these simulated values as necessary, simulate one set of interest rates, and calculate the EPV of the benefits and the EPV of annual premium P under these simulated interest rates.

   The insurance company simulates 10,000 sets of interest rates, and under its simulations the EPV of the benefits has mean \$3,144.65, and standard deviation \$85.73. The EPV of $a_{57:\overline{5}|}$ has mean 4.164824 and standard deviation 0.1077111. The covariance of the simuated EPV of benefits and the simulated value of $a_{57:\overline{5}|}$ is 9.230819.

   (b) Find a 95% confidence interval for the EPV of the benefits

(c) Find a 95% confidence interval for the net premium — that is the value of P such that the EPV of future loss is zero.

8. An insurance company sells a 5-year annual life insurance policy to a life aged 53, for whom the lifetable below is appropriate.

x	l_x	d_x
53	10000.00	49.24
54	9950.76	54.62
55	9896.14	60.60
56	9835.55	67.22
57	9768.32	74.56
58	9693.76	82.68

The annual gross premium is \$685. Initial expenses are \$400. The death benefits are \$90,000. Renewal costs are 2% of each subsequent premium. The interest rate is $i = 0.05$

(a) Calculate the profit vector for the policy.

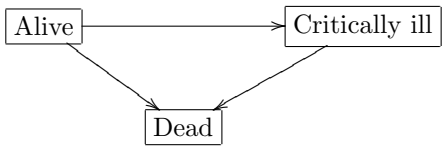
(b) Calculate the discounted payback period of the policy using a risk discount rate $i = 0.07$.

9. An insurance company sells a 5-year endowment insurance policy to a life aged 35 for whom the lifetable below is appropriate.

x	l_x	d_x
35	10000.00	8.74
36	9991.26	9.45
37	9981.81	10.24
38	9971.57	11.12
39	9960.45	12.11
40	9948.35	13.22

The benefit is \$300,000. The annual premium is \$60,000, and the interest rate is $i = 0.03$. Initial expenses are \$2,400 and renewal expenses are \$80 at the start of each year after the first. Use a profit test to calculate the reserves at the start of each year. There are no exits other than death or maturity.

10. An insurance company offers a 5-year critical illness insurance policy. The policy has 3 states — alive, critically ill, and dead. The possible transitions are as shown in the following diagram:



Premiums are payable at the start of each year while in the alive state.

For a life aged 37, transitions are as shown in the following lifetable:

age	Alive	Critically Ill	Death (direct)	Death (critically ill)	CI and Death
37	10000.00	0.00	6.95	0.00	0.03
38	9990.20	2.82	7.47	0.03	0.04
39	9979.47	6.01	8.03	0.08	0.03
40	9967.71	9.63	8.66	0.15	0.04
41	9954.78	13.75	9.36	0.22	0.03

At the end of 5 years, the expected number of lives who are critically ill is 18.42.

Initial expenses are 28% of the first premium, and renewal expenses are 4% of subsequent premiums while the life is in the alive state. There are also renewal expenses of \$80 at the start of each year if the life is in the critically ill state. Premiums are payable at the start of each year when the life is in the healthy state. There is a death benfit of \$250,000 at the end of the year in which the life dies, and a benefit of \$100,000 at the end of the year in which the life becomes critically ill. (If the life becomes critically ill and then dies later in the same year, both benefits are payable at the end of the year.) The interest rate is $i = 0.04$. Use a profit test without reserves to determine the premium for this policy which achieves a profit margin of 5% at a risk discount rate of $i = 0.10$.

11. A policyholder aged 58 buys a 5-year type B universal life insurance policy. The additional death benefit is \$100,000. The policyholder pays a premium of \$7,200 at the start of each year. The lifetable for the policyholder is:

x	l_x	d_x
58	10000.00	4.12
59	9995.88	4.41
60	9991.46	4.73
61	9986.73	5.08
62	9981.65	5.46
63	9976.19	5.87

The cost of insurance is based on 105% of mortality in the above table and $i = 0.04$. Expense charges are 1.5% of the account value (after each premium is paid). Assume the credited interest rate is $i = 0.05$.

(a) Calculate the projected account value for the next 5 years.

(b) Suppose the insurer earns an interest rate, $i = 0.08$, and mortality follows the above table, initial expenses are \$1,900 and renewal expenses are 0.5% of account value each year after the first. Suppose there are no surrenders. Calculate the profit margin of this policy at a risk discount rate of $i = 0.12$.

12. A life aged 62 buys a 5-year type A universal life insurance policy with death benefit \$600,000. The annual premium is \$7,600. Mortality is as shown in the following table:

x	l_x	d_x
62	10000.00	12.33
63	9987.67	13.28
64	9974.39	14.33
65	9960.06	15.48
66	9944.58	16.75
67	9927.83	18.15

The credited interest rate is $i = 0.08$. Cost of insurance is based on mortality in the above table and $i = 0.05$. Expense charges are 1.5% of account value (before applying cost of insurance).

(a) Project the account value for the next 5 years.

(b) Assume that the insurance company earns interest $i = 0.075$; Mortality follows the mortality in the lifetable; Initial expenses are \$2,020; renewal expenses are 0.6% of premiums paid. The surrender charges and surrender rates are:

Year	Charge	rate
1	\$3,000	4%
2	\$2,200	5%
3	\$1,500	4%
4	\$800	2%
5	0	100%

Calculate the NPV of the policy at a risk discount rate of 10%.

13. A life aged 48 has an annual type A Universal life insurance policy that has been in effect for 3 years.

- The current account value is \$38,220.
- The annual premium is \$6,800.
- The expense charge is 1% of account value.
- The credited interest rate is $i = 0.07$.
- The total death benefit is \$100,000.
- The corridor factor requirement is 2.2.
- The insurance is priced using mortality rate $q_{48} = 0.000216$ and interest $i = 0.04$.

Calculate the cost of insurance charge for the year.

14. An equity-linked insurance policy has the following properties:

- Annual premiums are \$7,000.
- Expense charges are 6% of the first premium and 1.5% of subsequent premiums.
- There is a year-end management fee of 0.7% of fund value.
- There is a year-end death benefit of 120% of fund value.
- Surrenders receive full fund value.
- GMMB is the total of the premiums paid.
- The annual return is 6%.
- The insurer's initial expenses are \$500 plus 7.2% of the first premium.
- The insurer's renewal expenses are 0.9% of each subsequent premium.
- Mortality is given by $q_x = 0.0002 + 0.00001x$.
- The policy is sold to a life aged 47.
- The policy matures in 5 years.
- Surrenders happen at a rate of 4% per year.

(a) Project the fund value for the next 5 years.

(b) Calculate the profit vector for the policy.

(c) If mortality and surrender rates are exactly as in the model, which of the following is the internal rate of return of the policy?

(i) 9.42%

(ii) 10.58%

(iii) 11.04%

(iv) 11.90%

15. For an equity-linked insurance policy with the following properties:

- Annual premiums are \$4,000.
- Expense charges are 10% of the first premium and 1.2% of subsequent premiums.
- There is a year-end management fee of 1% of fund value.
- There is a year-end death benefit of 130% of fund value.
- Surrenders receive full fund value.
- GMMB is the total of the premiums paid.
- The insurer's initial expenses are \$300 plus 20% of the first premium.
- The insurer's renewal expenses are 0.3% of each subsequent premium.
- Mortality is given by $q_x = 0.0002 + 0.00001x$.
- The policy is sold to a life aged 54.
- The policy matures in 5 years.
- Surrenders happen at a rate of 5% per year.

(a) Use the following random numbers from a standard normal distribution to simulate 10 years of annual returns following a log-normal distribution with $\mu = 0.04$ and $\sigma = 0.08$.

```
1.50367034  0.21416629  0.28936230  -0.32615940  1.66506629  0.02024787  -2.64251672  -0.78465126
-0.71987248  0.84399470
```

(b) Use the simulated returns to project the account values for the next 5 years.

(c) Calculate the NPV for the policy under these returns at a risk discount rate of $i = 10\%$.

16. An equity-linked insurance policy has the following properties:

- Annual premiums are \$11,000.
- Expense charges are 10% of the first premium and 1% of subsequent premiums.
- There is a year-end management fee of 1% of fund value.
- There is a year-end death benefit of 150% of fund value.
- Surrenders receive full fund value.
- GMMB is the total of the premiums paid.
- The insurer's initial expenses are \$400 plus 10% of the first premium.
- The insurer's renewal expenses are 0.5% of each subsequent premium.
- Mortality is given by $q_x = 0.0002 + 0.00003x$.
- The policy is sold to a life aged 44.

- The policy matures in 10 years.
- Surrenders happen at a rate of 2% per year.
- The insurers' funds receive an annual return of 5%.
- Annual returns are log-normally distributed with $\mu = 0.05$ and $\sigma = 0.09$.

The insurance company simulates 5000 sets of annual returns.

The expected fund value at the end of each year (after the management charge) is given in the following table.

Year	Expected fund value
1	\$10,777.76
2	\$23,738.14
3	\$36,545.19
4	\$50,891.99
5	\$69,002.40
6	\$87,803.32
7	\$105,490.06
8	\$126,203.20
9	\$145,938.26
10	\$168,093.55

In 233 of their simulations, the fund value at the end of year 10 was less than \$110,000. The mean fund value at the end of year 10 for these simulations was \$89,492.45. The policy has no reserves.

Calculate the NPV of this policy for the simulated returns at a risk discount rate of 10%.

17. An equity-linked insurance policy has the following properties

- Annual premiums are \$8,000.
- There is a year-end death benefit of 110% of fund value.
- Acquisition expenses are \$17,000.
- The insurer's renewal expenses are \$50 at the start of each year after the first.
- The policy matures in 15 years.
- Reserves on the policy receive an annual return of $i = 0.02$ and are calculated on a reserve basis of 110% of assumed mortality, annual returns of $i = 0.02$ on the fund.

They plan to use simulation to determine the GMMB, expenses and management charge for the policy. They plan to arrange these so that the NPV of the policy at a risk discount rate of 10% is at least 50% of the acquisition costs, and the probability of making a loss at the risk discount rate is at most 2%. The insurance company simulates 10000 sets of investment returns.

(a) For the first simulation, the company finds that even with the management charge increasing to 100%, the probability of making a loss is still more than 2%. Which of the following changes might solve this problem:

(i) Increasing the expense charges

(ii) Decreasing the expense charges

(iii) Increasing the GMMB

(iv) Decreasing the GMMB

(b) The insurance company finds a combination of charges and benefits which achieves its criteria. However, a new manager suggests that the risk discount rate should be increased to 15%. They are surprised to find that this reduces the management charge required to satisfy the criteria. Why does this happen, and what should be done (if anything) to correct this situation?

(c) The insurance company decides to switch to using a simulation to calculate a 95% quantile reserve in place of the reserves calculated above. If each quantile reserve is to be calculated based on 1,000 simulations, and the overall policy charges and benefits are to be calculated based on 5,000 simulations, how many annual returns do they need to simulate to achieve this? Explain your answer.

18. An equity-linked insurance policy has the following properties

- Annual premiums are \$5,000.
- Expense charges are 0.8% of premiums after the first.
- There is a year-end management fee of 1.2% of fund value.
- There is a year-end death benefit of 130% of fund value.
- GMMB is 110% of the total of the premiums paid.
- The insurer's renewal expenses are 0.4% of each subsequent premium.
- The policy matures in 10 years.
- Annual returns are log-normally distributed with $\mu = 0.05$ and $\sigma = 0.3$.
- Mortality is given by $q_x = 0.0002 + 0.00003x$.
- Reserves on the policy receive an annual return of $i = 0.02$.

The policy was sold 9 years ago to a life aged 51. The fund value at the beginning of Year 10 (before premiums are received) is \$41,205. The company simulates 1000 rates of return. The simulated returns have the following percentiles.

Quantile	i
2.5%	-0.1377533
5%	-0.1010528
95%	0.2395570
97.5%	0.2840839

Calculate a 95% quantile reserve for the policy at the start of year 10, based on these simulated values.