ACSC/STAT 4720, Life Contingencies II
Fall 2017

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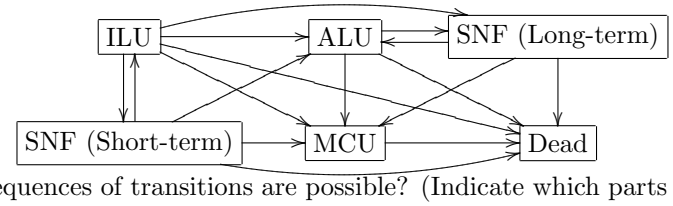
Homework Sheet 1

Due: Friday 29th September: 12:30 PM

Basic Questions

1. An CCRC is developing a model for its care costs. The community has four levels of care: Independent Living Unit, Assisted Living Unit, Skilled Nursing Facility, and Memory Care Unit. The transition diagram is shown below:



Which of the following sequences of transitions are possible? (Indicate which parts of the transition sequence are not possible if the sequence is not possible.)

(i) ILU—SNF (short-term)— ALU—Dead

(ii) ILU—ALU—SNF (long-term)—ILU

(iii) ILU—ALU—MCU—Dead

(iv) ILU—SNF (long-term)—MCU—ALU

(v) ILU—MCU—ALU—Dead

2. Consider a permanent disability model with transition intensities

$$\mu_x^{01} = 0.004 + 0.000001x$$
$$\mu_x^{02} = 0.001 + 0.000005x$$
$$\mu_x^{12} = 0.002 + 0.000003x$$

where State 0 is healthy, State 1 is permanently disabled and State 2 is dead.

(a) Calculate the probability that a healthy individual aged 22 is still healthy at age 41.

(b) Calculate the probability that a healthy individual aged 22 is dead by age 38.

3. Under a disability income model with transition intensities

$$\mu_x^{01} = 0.001$$
$$\mu_x^{10} = 0.002$$
$$\mu_x^{02} = 0.003$$
$$\mu_x^{12} = 0.005$$

calculate the probability that a healthy individual dies within the next 4 years. [State 0 is healthy, State 1 is sick and State 2 is dead.]

4. Under a critical illness model with transition intensities

$$\begin{aligned}\mu_x^{01} &= 0.001\\ \mu_x^{02} &= 0.002\\ \mu_x^{12} &= 0.12\end{aligned}$$

calculate the premium for a 10-year policy with premiums payable continuously while the life is in the healthy state, which pays a death benefit of \$130,000 upon entry into state 2, and a benefit of \$80,000 upon entry into state 1, sold to a life in the healthy state (state 0). The interest rate is $\delta = 0.06$ [State 0 is healthy, State 1 is sick and State 2 is dead.]

5. An employer offers a survivor benefit insurance policy. The possible exits from this policy are retirement, surrender, and death. The transition intensities are

$$\begin{aligned}\mu_x^{01} &= 0.002 + 0.000003x\\ \mu_x^{03} &= 0.001 + 0.000004x\\ \mu_x^{02} &= \begin{cases} 0 & \text{if } x < 60\\ 0.2(x-60) & \text{if } x \geqslant 60\end{cases}\end{aligned}$$

Calculate the probability that an individual aged 34 withdraws from the policy before age 64. [State 0 is healthy, State 1 is surrender, State 2 is retired and State 3 is dead.]

Standard Questions

6. An insurance company is developing a new model for transition intensities in a disability income model. Under these transition intensities it calculates

$$\begin{array}{lll}\overline{a}_{27}^{00} = 18.17 & \overline{a}_{37}^{00} = 17.83 & \overline{a}_{37}^{10} = 0.98\\ \overline{a}_{27}^{01} = 0.84 & \overline{a}_{37}^{01} = 0.73 & \overline{a}_{37}^{11} = 15.42\\ {}_{10}p_{27}^{00} = 0.919 & {}_{10}p_{27}^{01} = 0.026 & \delta = 0.05\end{array}$$

Calculate the premium for a 10-year policy for a life aged 27, with continuous premiums payable while in the healthy state, which pays a continuous benefit while in the sick state, at a rate of \$80,000 per year, and pays a death benefit of \$900,000 immediately upon death. [Hint: to calculate A_x^{02}, consider how to extend the equation $\overline{a}_x = \frac{1-\overline{A}_x}{\delta}$ to the multiple state case by combining states 0 and 1.]