ACSC/STAT 4720, Life Contingencies II
Fall 2018

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Homework Sheet 3

Due: Friday 12th October: 14:30 PM

Basic Questions

1. A life aged 54 wants to buy a 5-year term insurance policy. A life-table based on current-year mortality is:

x	l_x	d_x
54	10000.00	6.92
55	9993.08	7.62
56	9985.47	8.40
57	9977.07	9.26
58	9967.82	10.21

The insurance company uses a single-factor scale function $q(x,t) = q(x,0)(1-\phi_x)^t$ to model changes in mortality. The insurance company uses the following values for ϕ_x:

x	ϕ_x
54	0.02
55	0.02
56	0.025
57	0.015
58	0.02

Calculate $A^{1}_{54:\overline{5}|}$ at interest rate $i = 0.04$, taking into account the change in mortality.

2. Using the lifetable from Question 1, the insurance company now uses the following mortality scale based on both age and year:

	t					
x	2018	2019	2020	2021	2022	2023
54	0.02	0.015	0.01	0.01	0.02	0.015
55	0.01	0.02	0.025	0.02	0.015	0.02
56	0.02	0.025	0.03	0.025	0.02	0.015
57	0.025	0.02	0.02	0.015	0.01	0.015
58	0.015	0.015	0.025	0.03	0.035	0.025

Use this mortality scale to calculate $A^{1}_{54:\overline{5}|}$ at interest rate $i = 0.04$.

3. A pensions company has the current mortality scale for 2018:

x	$\phi(x, 2018)$	$\frac{d\phi(x,t)}{dt}\Big\vert_{x,t=2018}$	$\frac{d\phi(x+t,t)}{dt}\Big\vert_{x,t=2018}$
54	0.015845470	−0.001202552	0.0029517451
55	0.006067218	−0.003357078	0.0002835208
56	0.019612949	−0.003639662	−0.0043587062
57	0.024808173	−0.007938091	0.0005934601
58	0.012475802	−0.003159578	−0.0003959116

Current mortality is given in the lifetable in Question 1. The company assumes that from 2030 onwards, we will have $\phi(x, t) = 0.02$ for all x and t. Calculate $A^{1}_{54:\overline{5}|}$ at interest rate $i = 0.04$, using the average of age-based and cohort-based effects.

Standard Questions

4. An insurance company uses a Lee-Carter model and fits the following parameters:

$$c = -0.8 \qquad \sigma_k = 1.3 \qquad K_{2018} = -4.14$$

And the following values of α_x and β_x:

x	α_x	β_x
53	−4.180251	0.1791691
54	−4.219389	0.1788574
55	−4.320727	0.1780642
56	−4.080177	0.1799758
57	−4.397765	0.1790583
58	−4.008800	0.1836070
59	−4.424434	0.1794805
60	−4.354352	0.1812529

The insurance company simulates the following values of Z_t:

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0.48683324 -0.69007524 -1.34565369 -0.44229856 -0.01575498 -0.38189150 1.57336437
-0.69746487
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Using these simulated values, calculate the probability that a life aged exactly 53 at the start of 2018 survives for 8 years.