ACSC/STAT 4720, Life Contingencies II
Fall 2018
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Homework Sheet 7
Due: Friday 30th November: 12:30 PM

Basic Questions

1. An insurance company sells a 5-year annual life insurance policy to a life aged 36, for whom the lifetable below is appropriate.

x	l_x	d_x
36	10000.00	9.64
37	9990.36	10.13
38	9980.23	10.68
39	9969.55	11.30
40	9958.25	11.98

The annual gross premium is $625.40. Initial expenses are $190 plus 40% of the first premium. The death benefits are $500,000. Renewal costs are 3% of each subsequent premium. The interest rate is $i = 0.04$

(a) Calculate the expected net cash-flows associated with this policy (assuming no reserve). [This is the profit vector for the policy.]

(b) Which of the following is the internal rate of return of the policy:

(i) $i = 0.0940$

(ii) $i = 0.1244$

(iii) $i = 0.1576$

(iv) $i = 0.1694$

2. An insurance company sells a 5-year annual life insurance policy to a life aged 40, for whom the lifetable below is appropriate.

x	l_x	d_x
40	10000.00	6.63
41	9993.37	7.37
42	9986.00	8.20
43	9977.80	9.12
44	9968.68	10.16

The annual gross premium is $280. Initial expenses are $80 plus 25% of the first premium. The death benefits are $300,000. Renewal costs are 2% of each subsequent premium. The interest rate is $i = 0.05$. Reserves are calculated on the basis $i = 0.03$, with mortality following the table.

(a) Calculate the reserves.

(b) Calculate the profit signature.

(c) Calculate the profit margin at a risk discount rate of $i = 0.08$.

3. For the policy in Question 2:

   (a) Calculate the reserves and profit signature for a general premium. [You may assume that P is such that the reserves are zero in Years 1, 2 and 3.]

   (b) Calculate the premium that gives an internal rate of return of $i = 0.14$.

4. For a 5-year term insurance policy sold to a life aged 52, with the following lifetable:

x	l_x	d_x
52	10000.00	30.46
53	9969.54	33.90
54	9935.64	37.72
55	9897.92	41.96
56	9855.95	46.67

   an actuary performs the following profit test without reserves:

Year	Premium	Expenses	Interest	Expected Death Benefits	Pr_t
0		200			-200
1	1700	0	102.00	1370.70	431.30
2	1700	54	99.24	1530.16	215.08
3	1700	54	99.24	1708.40	36.84
4	1700	54	99.24	1907.67	-162.43
5	1700	54	99.24	2130.84	-385.60

   Calculate the reserves needed to ensure that all cash flows are non-negative.

Standard Questions

5. An insurer sells a 5-year disability income protection policy for a life aged 48. The transition probabilities are given in the following table:

x	p_x^{01}	p_x^{02}	p_x^{10}	p_x^{12}
48	0.002032	0.001362	0.13453	0.06150
49	0.002143	0.001481	0.13264	0.07404
50	0.002305	0.001635	0.13007	0.08787
51	0.002521	0.001810	0.12635	0.10318
52	0.002730	0.002024	0.12395	0.12466

   The probability of being in each state at the end of each year is

t	$_tp_{48}^{00}$	$_tp_{48}^{01}$
1	0.996606	0.001362
2	0.993174955536	0.002556475326
3	0.989594366957	0.00362315814575
4	0.985766219786	0.00458270046075
5	0.981647912899	0.00543858612805

   The policy pays a benefit of $35,000 at the end of any year if the life is disabled at that time (State 1), and pays a death benefit of $540,000 at the end of the year if the life is dead (State 2). The interest rate is $i = 0.04$. Initial expenses are $600 plus 30% of the first premium. Renewal expenses are 2.5% of each

subsequent premium. The premium is \$1,960 at the start of each year. Use a profit test to calculate the reserves for each year in each state using a reserve rate of $i = 0.05$ and calculate the profit margin at a risk discount rate of $i = 0.12$.