ACSC/STAT 4720, Life Contingencies II

FALL 2021

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Homework Sheet 2

Due: Thursday 7th October: 14:30

Basic Questions

1. The following is a standard multiple decrement table giving probabilities of death (decrement 1) and surrender (decrement 2) for a life insurance policy:

x	l_x	$d_x^{(1)}$	$d_x^{(2)}$
51	10000.00	19.48	3.58
52	9976.94	22.19	3.48
53	9951.27	25.24	3.38
54	9922.64	28.64	3.31
55	9890.70	32.44	3.25

A life who is in poor health has the following lifetable.

x	l_x	d_x
51	10000.00	443.73
52	9556.27	509.55
53	9046.72	579.76
54	8466.95	652.25
55	7814.70	723.74

Use this lifetable and the standard multiple decrement table to produce a multiple decrement table for this life, assuming that this life has standard surrender probabilities, using:

(a) UDD in the multiple decrement table.

(b) UDD in the independent decrements.

2. The mortalities for a husband and wife (whose lives are assumed to be independent) aged 35 and 62 respectively, are given in the following tables:

x	l_x	d_x
35	10000.00	25.33
36	9974.67	26.90
37	9947.77	28.60
38	9919.17	30.42
39	9888.75	32.38
40	9856.37	34.49
41	9821.88	36.76
42	9785.11	39.20
43	9745.91	41.82
44	9704.08	44.64
45	9659.45	47.65

x	l_x	d_x
62	10000.00	110.82
63	9889.18	117.39
64	9771.79	124.26
65	9647.53	131.42
66	9516.11	138.87
67	9377.24	146.61
68	9230.63	154.62
69	9076.00	162.89
70	8913.11	171.40
71	8741.71	180.13
72	8561.58	189.04

The interest rate is $i = 0.04$.

(a) They want to purchase an 8-year joint life insurance policy with a death benefit of \$1,200,000. Annual premiums are payable while both are alive. Calculate the net premium for this policy using the equivalence principle.

(b) They want to purchase a 9-year last survivor insurance with a benefit of \$12,000,000. Premiums are payable while either life is alive. Calculate the net premium for this policy using the equivalence principle.

3. A husband is 76; the wife is 41. Their lifetables while both are alive, and the lifetable for the wife if the husband is dead, are given below:

x	l_x	d_x
76	10000.00	1473.82
77	8526.18	1409.36
78	7116.83	1319.45
79	5797.37	1205.58
80	4591.79	1071.07
81	3520.72	921.20

x	l_x	d_x
41	10000.00	14.70
42	9985.30	15.94
43	9969.37	17.30
44	9952.07	18.80
45	9933.27	20.44
46	9912.83	22.25

x	l_x	d_x
41	10000.00	161.87
42	9838.13	186.25
43	9651.88	213.77
44	9438.12	244.61
45	9193.51	278.89
46	8914.62	316.59

Calculate the probability that the wife is alive in 5 years time. Use the UDD assumption for handling changes to the wife's mortality in the year of the husband's death.

Standard Questions

4. The following is a multiple decrement table giving probabilities of surrender (decrement 1) and death (decrement 2) for a life insurance policy:

x	l_x	$d_x^{(1)}$	$d_x^{(2)}$
44	10000.00	21.36	6.74
45	9971.90	19.85	11.25
46	9940.80	18.47	15.95
47	9906.39	17.21	20.89
48	9868.29	16.08	26.10
49	9826.11	15.04	31.65
50	9779.42	14.10	37.59
51	9727.73	13.24	43.97
52	9670.52	12.46	50.83
53	9607.23	11.75	58.25
54	9537.23	11.10	66.28

   A life insurance policy pays a benefit of \$640,000 at the end of the year of death. Premiums are payable at the beginning of each year. Calculate the premium for a 10-year policy sold to a life aged 44 if the interest rate is $i = 0.08$.

5. A couple want to receive the following:

   - While both are alive, they would like to receive a pension of \$140,000 per year.
   - If the husband is alive and the wife is not, they would like to receive a pension of \$90,000 per year.
   - If the wife is alive and the husband is not, they would like to receive a pension of \$70,000 per year.
   - When the husband dies: if the wife is still alive, they would like a death benefit of \$700,000; otherwise, they would like a death benefit of \$300,000.
   - When the wife dies: if the husband is still alive, they would like a death benefit of \$400,000; otherwise, they would like a death benefit of \$200,000.

   Construct a combination of insurance and annuity policies that achieve this combination of benefits.

6. A husband aged 61 and wife aged 54 have the following transition intensities:

$$\mu_{xy}^{01} = 0.005y - 0.09$$
$$\mu_{xy}^{02} = 0.003x - 0.027$$
$$\mu_{xy}^{03} = 0.08$$
$$\mu_{x}^{13} = 0.006x - 0.022$$
$$\mu_{y}^{23} = 0.012y - 0.024$$

They want to purchase a last survivor insurance, which will pay a benefit of \$1,700,000 when the second life dies. Premiums are payable continuously while either life is alive. Force of interest is $\delta = 0.04$.

(a) Calculate the annual rate of continuous premium.

(b) Calculate the policy value after 3 years if the husband is dead and the wife is alive.