ACSC/STAT 4720, Life Contingencies II

FALL 2021
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Homework Sheet 3

Due: Thursday 14th October: 14:30

Basic Questions

1. A life aged 62 wants to buy a 5-year term insurance policy. A life-table based on current-year (2021) mortality is:

x	l_x	d_x
62	10000.00	157.11
63	9842.89	167.55
64	9675.34	178.46
65	9496.87	189.81
66	9307.06	201.57

The insurance company uses a single-factor scale function $q(x,t) = q(x,0)(1-\phi_x)^t$ to model changes in mortality. The insurance company uses the following values for ϕ_x:

x	ϕ_x
62	0.01
63	0.02
64	0.03
65	0.01
66	0.01

Calculate $\ddot{a}_{62:\overline{5}|}$ at interest rate $i = 0.06$, taking into account the change in mortality.

2. Using the lifetable from Question 1, the insurance company now uses the following mortality scale, $\phi(x,t)$ based on both age and year:

	t				
x	2022	2023	2024	2025	2026
62	0.015	0.015	0.020	0.015	0.020
63	0.045	0.000	0.005	0.020	0.015
64	−0.020	0.005	0.005	0.025	0.010
65	0.025	0.005	0.030	0.015	0.010
66	0.025	0.015	0.040	0.010	0.025

Use this mortality scale to calculate $A^{1}_{62:\overline{5}|}$ at interest rate $i = 0.05$.

3. A life-insurance company has the current mortality scale for 2021:

x	$\phi(x, 2022)$	$\left.\frac{d\phi(x,t)}{dt}\right\rvert_{x,t=2022}$	$\left.\frac{d\phi(x+t,t)}{dt}\right\rvert_{x,t=2022}$
62	0.019716147074	0.0010174563604	−0.0072041156604
63	0.002020553601	−0.0034265947953	0.0026162132756
64	0.006613716415	−0.0003726756896	0.0027308379647
65	0.007275748793	0.0002926793710	−0.0009361061799
66	0.002408521108	−0.0019393709894	0.0007201245263

Current mortality (in 2021) is given in the lifetable in Question 1. The company assumes that from 2031 onwards, we will have $\phi(x, t) = 0.01$ for all x and t. Calculate $\ddot{a}_{62:\overline{5}|}$ at interest rate $i = 0.05$, using the average of age-based and cohort-based effects.

Standard Questions

4. An insurance company uses a Lee-Carter model and fits the following parameters:

$$c = -0.65 \qquad \sigma_k = 1.4 \qquad K_{2021} = -3.29$$

And the following values of α_x and β_x:

x	α_x	β_x
42	−3.445547529	0.2160196693
43	−3.723003508	0.2056043631
44	−3.240526315	0.2319018119
45	−3.213960546	0.2160218805
46	−3.394213139	0.2669114067
47	−3.014411418	0.2324790526
48	−3.275815282	0.2361910612

The insurance company simulates the following values of Z_t:

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-0.8654056910 -0.9142362784 -1.2831326166 1.0005379227
0.3053339512 0.1684182795 -0.1596511482
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Using these simulated values, calculate the probability that a life aged exactly 42 at the start of 2021 survives for 6 years.