ACSC/STAT 4720, Life Contingencies II

FALL 2021

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Homework Sheet 4

Due: Thursday 21st October: 14:30

1. An insurance company uses a Lee-Carter model and fits the following parameters:

$$c = -0.3 \quad \sigma_k = 1.2 \quad K_{2021} = -2.25 \quad \alpha_{36} = -1.74 \quad \beta_{36} = 1.11$$

It estimates that its reserves are adequate in a given year provided $q(36,t) < 0.0034$. Calculate the probability that its reserves are still adequate in 7 years' time. Use UDD to calculate the relation between q_x and m_x.

2. An insurance company uses a Cairns-Blake-Dowd model with the following parameters:

$$K^{(1)}_{2021} = -9.37 \quad K^{(2)}_{2021} = 0.11 \quad c^{(1)} = -0.12 \quad c^{(2)} = 0.02$$
$$\sigma_{k_1} = 0.7 \quad \sigma_{k_2} = 0.06 \quad \rho = 0.4 \quad \overline{x} = 46$$

(a) Use this scale to calculate the median value of $q(33, 2029)$.

(b) A life aged 72 will only be approved for life insurance if her mortality is less than 0.1. How long can she wait to purchase a life insurance contract and still have a 70% probability of being approved? [Remember that her age also increases by 1 each year.]

Standard Questions

3. An insurance company uses a Lee-Carter model and fits the following parameters:

$$c = -0.5 \qquad \sigma_k = 1.3 \qquad K_{2021} = -5.12$$

And the following values of α_x and β_x:

x	α_x	β_x
33	-6.788236	0.2228085
34	-6.750172	0.1375526
35	-6.755374	0.1979110
36	-6.720697	0.1529246
37	-6.694897	0.2131581

Using the approximation $m(x,t) \approx q(x,t)$, calculate the probability that a life aged 33 dies at age 35 under this model.

4. An insurance company uses a Cairns-Blake-Dowd model with the following parameters:

$$K_{2021}^{(1)} = -4.33 \qquad K_{2021}^{(2)} = 0.22 \qquad c^{(1)} = -0.15 \qquad c^{(2)} = 0.01$$
$$\sigma_{k_1} = 0.8 \qquad \sigma_{k_2} = 0.06 \qquad \overline{x} = 53$$

It has not yet decided on a suitable value of ρ. The company sells both life insurance and annuity contracts. It's expected profit in 2024 (in millions) is $36 + 22.4q(58, 2024) - 38.8q(73, 2024)$. In order to satisfy the regulators, it needs to ensure that the expected profit has a 95% probability of being positive. For what values of ρ is this achieved?

(i) $\rho < 0.24$

(ii) $\rho > 0.24$

(iii) $\rho < 0.33$

(iv) $\rho > 0.33$

(v) $\rho < 0.45$

(vi) $\rho > 0.45$

(vii) $\rho < 0.52$

(viii) $\rho > 0.52$

Justify your answer. [You may need to use simulation to numerically estimate the probability of profit.]