# MATH 3090, Advanced Calculus I
## Fall 2006
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### Homework Sheet 7
### Due in: Wednesday 15th November, 11:30 AM

Please hand in solutions to questions 1-3.

## Compulsory questions

1 Find the Fourrier coefficients for the following functions. You may use either $f(x)=\sum_{n=-\infty}^{\infty} c_n e^{inx}$ or $f(x)=\frac{1}{2}a_0+\sum_{n=1}^{\infty} a_n \cos nx + b_n \sin nx$.

(a) $f(x)=\begin{cases} -1 & \text{if } (2n-1)\pi < x \leqslant 2n\pi \\ 1 & \text{if } 2n\pi < x \leqslant (2n+1)\pi \end{cases}$ for any integer $n$.

(b) $f(x)=y^3$ where $y$ is the value of $x-2n\pi$ with smallest modulus, and $f((2n+1)\pi)=0$ (so $f(x)=x^3$ for $-\pi < x < \pi$, and $f$ is $2\pi$-periodic).

2 (a) Show that $f(x)$ given by $f(x)=x^4$ on $[-\pi,\pi)$, and $f$ is $2\pi$-periodic, has Fourier series $f(x)=\frac{\pi^4}{10}+\sum_{n=1}^{\infty}(-1)^n 4\left(\frac{\pi^2}{n^2}-\frac{6}{n^4}\right)\cos nx$.

(b) Deduce that $\sum_{n=1}^{\infty}\frac{(-1)^n}{n^4}=-\frac{7\pi^4}{720}$ (You may assume that $\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}=-\frac{\pi^2}{12}$).

3 Suppose $f(x)$ is a $2\pi$-periodic function with Fourier series $f(x)=\frac{1}{2}a_0+\sum_{n=0}^{\infty} a_n\cos((4n+1)x)$. Express the function $g(x)$ whose Fourier series is $g(x)=\frac{1}{2}a_0\cos x-\frac{1}{2}a_0\sin x+\sum_{n=0}^{\infty} a_n\cos((4n+2)x)$ in terms of $f(x)$. [Hint: $\cos((4n+2)x)=\cos x\cos((4n+1)x)-\sin x\sin((4n+1)x)$, and $\sin((4n+1)x)=\cos(4n+1)\left(x-\frac{\pi}{2}\right)$.]