# MATHEMATICS 1115A 2005 AUTUMN TEST 1

**SECTION _____ NAME (PRINTED) _____**

**Student Number _____ SIGNATURE _____**

***Only calculators without memory or graphics are allowed and no other electronic devices are allowed.*** **Each multiple choice question is worth 1 mark. All work is to be shown on this question paper in the space provided and *the answer selected must be entered on the attached answer sheet.* Both the question paper and the answer sheet, *both* with completed identification sections, must be handed in.**

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Suppose that an amount $P$ is invested at a rate of 12% compounded monthly.

1. If we want to know how long it will take the investment to double in value, we must solve:

(A) $2P = P(1.12)^n$ for $n$ (B) $2P = P(1.01)^n$ for $P$ (C) $2P = P(1.12)^n$ for $P$
(D) $2P = P(1 + \frac{12}{12})^n$ for $n$ (E) $2P = P(1.01)^n$ for $n$

2. Solving the equation gives:

(A) $n = \ln 2 - \ln(1.01)$ (B) $P = \dfrac{\ln 2}{\ln(1.12)}$ (C) $n = \dfrac{\ln 2}{\ln(1 + \frac{12}{12})}$
(D) $P = \dfrac{\ln 2}{\ln(1.01)}$ (E) $n = \dfrac{\ln 2}{\ln(1.01)}$

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An initial investment of \$25,000 in a business guarantees the following cash flows:

| Year | Cash Flow |
|---|---|
| 3 | \$8,000 |
| 4 | \$10,000 |
| 6 | \$14,000 |

The interest available is 6% compounded semi-annually.

3. The net present value (NPV) of the cash flows is given by:

(A) $8000(1.03)^{-6} + 10,000(1.03)^{-8} + 14,000(1.03)^{-12} + 25,000$
(B) $8000(1.03)^{-6} + 10,000(1.03)^{-8} + 14,000(1.03)^{-12} - 25,000$
(C) $8000(1.03)^{6} + 10,000(1.03)^{8} + 14,000(1.03)^{12} - 25,000$
(D) $8000(1.06)^{-3} + 10,000(1.06)^{-4} + 14,000(1.06)^{-6} - 25,000$
(E) $8000(1.06)^{3} + 10,000(1.06)^{4} + 14,000(1.06)^{6} + 25,000$

4. The investment is:

(A) profitable, if the answer to question 3. is less than 0 (B) profitable, in any event
(C) profitable, if the answer to question 3. is less than -1
(D) unprofitable, if the answer to question 3. is less than 0
(E) unprofitable, if the answer to question 3. is less than 1

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Recall that the present value of an (ordinary) annuity of $R$ dollars per payment period for $n$ periods at the interest rate of $r$ per period is given by

$$A = Ra_{\overline{n}|r} \quad \text{where} \quad a_{\overline{n}|r} = \frac{1-(1+r)^{-n}}{r}$$

and the future value is given by

$$S = Rs_{\overline{n}|r} \quad \text{where} \quad s_{\overline{n}|r} = \frac{(1+r)^{n}-1}{r}$$

An automobile loan of \$8500 is to be amortized over 48 months at an interest rate of 13.2% (per year) compounded monthly and payments are to be made each month.

5. The amount of each payment is given by:

(A) $\dfrac{8500}{a_{\overline{48}|0.011}}$ (B) $\dfrac{8500}{48}(1.011)$ (C) $\dfrac{8500}{s_{\overline{48}|0.011}}$ (D) $8500a_{\overline{48}|0.011}$ (E) $8500s_{\overline{48}|0.011}$

6. The correct expression in 5. can be evaluated by:

(A) $\dfrac{8500(0.011)}{1-(1.011)^{-48}}$ (B) $\dfrac{8500(0.011)}{(1.011)^{48}-1}$ (C) $\dfrac{8500(1-(1.011)^{-48})}{0.011}$
(D) $\dfrac{8500((1.011)^{48}-1}{0.011}$ (E) none of these

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7. If the matrix $\mathbf{A} = \begin{bmatrix} 2 & 4 & 6 & 8 \end{bmatrix}$ then the transpose of $\mathbf{A}$, $\mathbf{A}^T$, is:

(A) $\begin{bmatrix} 8 & 6 & 4 & 2 \end{bmatrix}$ (B) $\begin{bmatrix} -8 & -6 & -4 & -2 \end{bmatrix}$ (C) $\begin{bmatrix} 2 \\ 4 \\ 6 \\ 8 \end{bmatrix}$ (D) $\begin{bmatrix} 8 \\ 6 \\ 4 \\ 2 \end{bmatrix}$ (E) $\begin{bmatrix} -2 \\ -4 \\ -6 \\ -8 \end{bmatrix}$

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8. If $\mathbf{A} = \begin{bmatrix} 2 & 1 \\ 3 & -3 \end{bmatrix}$, $\mathbf{B} = \begin{bmatrix} -6 & -5 \\ 2 & -3 \end{bmatrix}$, and $\mathbf{C} = \begin{bmatrix} -2 & -1 \\ -3 & 3 \end{bmatrix}$ then $\mathbf{A} - \mathbf{B} + \mathbf{C} =$

(A) $\begin{bmatrix} -6 & 5 \\ -2 & 3 \end{bmatrix}$ (B) $\begin{bmatrix} -6 & -5 \\ -2 & -3 \end{bmatrix}$ (C) $\begin{bmatrix} 6 & 5 \\ 2 & 3 \end{bmatrix}$ (D) $\begin{bmatrix} -6 & -5 \\ 2 & -3 \end{bmatrix}$ (E) $\begin{bmatrix} 6 & 5 \\ -2 & 3 \end{bmatrix}$

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9. If $\mathbf{A} = \begin{bmatrix} 1 & 2 \\ 0 & -1 \\ 7 & 0 \end{bmatrix}$ and $\mathbf{D} = \begin{bmatrix} 1 & 2 & -1 \\ 1 & 0 & 2 \end{bmatrix}$ then $(\mathbf{D} - 2\mathbf{A}^T)^T =$

(A) $\begin{bmatrix} -1 & 2 & -15 \\ -3 & 2 & 2 \end{bmatrix}$ (B) $\begin{bmatrix} -1 & -3 \\ 2 & -2 \\ -15 & -2 \end{bmatrix}$ (C) $\begin{bmatrix} -1 & 2 & -15 \\ -3 & -2 & 2 \end{bmatrix}$
(D) none of these (E) $\begin{bmatrix} -1 & -3 \\ 2 & 2 \\ -15 & 2 \end{bmatrix}$

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10. If $\mathbf{A}$ is $2 \times 3$, $\mathbf{B}$ is $3 \times 3$, and $\mathbf{C}$ is $3 \times 2$ then $\mathbf{A}(\mathbf{B}^T)^2\mathbf{C}$ is:

(A) $2 \times 3$ (B) $2 \times 2$ (C) $3 \times 2$ (D) $3 \times 3$ (E) not defined

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11. $\begin{bmatrix} 1 & 0 & 6 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \end{bmatrix} =$

(A) $\begin{bmatrix} 0 & 0 & 12 & 6 \end{bmatrix}$ (B) [18] (C) $\begin{bmatrix} 0 \\ 0 \\ 12 \\ 6 \end{bmatrix}$ (D) undefined (E) $\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 12 & 0 \\ 0 & 0 & 0 & 6 \end{bmatrix}$

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12. If $\mathbf{B} = \begin{bmatrix} 0 & 2 & 0 \\ 0 & -1 & 0 \\ -1 & 0 & 2 \end{bmatrix}$ then $\mathbf{B}^2 =$

(A) $\begin{bmatrix} 0 & 4 & 0 \\ 0 & 1 & 0 \\ 4 & 4 & 4 \end{bmatrix}$ (B) $\begin{bmatrix} 0 & -2 & 0 \\ 0 & 1 & 0 \\ -2 & -2 & 4 \end{bmatrix}$ (C) $\begin{bmatrix} 0 & 2 & 0 \\ 0 & -1 & 0 \\ -1 & 0 & 2 \end{bmatrix}$ (D) $\begin{bmatrix} 0 & 4 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 4 \end{bmatrix}$ (E) $\begin{bmatrix} 0 & 0 & 1 \\ 4 & 1 & 0 \\ 0 & 0 & 4 \end{bmatrix}$

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13. The matrix $\begin{bmatrix} 2 & 3 \\ 1 & -6 \\ 4 & 8 \\ 1 & 7 \end{bmatrix}$ reduces to:

(A) $\begin{bmatrix} 1 & -6 \\ 0 & 1 \\ 0 & 32 \\ 0 & 13 \end{bmatrix}$ (B) $\begin{bmatrix} 2 & 3 \\ 1 & -6 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$ (C) $\begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$ (D) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$ (E) none of these

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The next two questions concern the system $\begin{cases} x + 3y + 2z - 1 = 0 \\ x + y + 5z - 10 = 0 \end{cases}$

14. The augmented matrix of the system reduces to:

(A) none of these (B) $\left[\begin{array}{ccc|c} 1 & 0 & \frac{13}{2} & \frac{29}{2} \\ 0 & 0 & 0 & 1 \end{array}\right]$ (C) $\left[\begin{array}{ccc|c} 1 & 0 & 0 & \frac{29}{2} \\ 0 & 0 & 1 & -\frac{9}{2} \end{array}\right]$ (D) $\left[\begin{array}{ccc|c} 1 & 0 & \frac{13}{2} & \frac{29}{2} \\ 0 & 1 & -\frac{3}{2} & -\frac{9}{2} \end{array}\right]$
(E) $\left[\begin{array}{ccc|c} 1 & 0 & 0 & \frac{29}{2} \\ 0 & 1 & 0 & -\frac{9}{2} \end{array}\right]$

15. The solutions of the system are:

(A) $(\frac{29}{2} - \frac{13}{2}z, -\frac{9}{2} + \frac{3}{2}z, z)$for any real number $z$ (B) none, last equation is $0 = 1$
(C) $(\frac{29}{2}, -\frac{9}{2}, z)$ for any real number $z$ (D) $(\frac{29}{2}, y, -\frac{9}{2})$ for any real number $y$ (E) none of these

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The next two questions concern the system $\begin{cases} 3w - x + 12y + 18z = -4 \\ w - 2x + 4y + 11z = -13 \\ w + x + 4y + 2z = 8 \end{cases}$

16. The augmented matrix of the system reduces to:

(A) $\left[\begin{array}{cccc|c} 1 & 0 & 4 & 5 & 1 \\ 0 & 1 & 0 & -3 & 7 \\ 0 & 0 & 0 & 0 & 1 \end{array}\right]$ (B) $\left[\begin{array}{cccc|c} 1 & 0 & 0 & 5 & 1 \\ 0 & 1 & 0 & -3 & 7 \\ 0 & 0 & 1 & 0 & 0 \end{array}\right]$ (C) $\left[\begin{array}{cccc|c} 1 & 0 & 4 & 5 & 1 \\ 0 & 1 & 0 & -3 & 7 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$ (D) $\left[\begin{array}{cccc|c} 1 & 0 & 4 & 0 & 1 \\ 0 & 1 & 0 & 0 & 7 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right]$
(E) none of these

17. The solutions of the system are:

(A) no solutions (B) $w = 1 - 5z$, $x = 7 + 3z$, $y = 0$, $z = z$
(C) $w = 1 - 4y$, $x = 7$, $y = y$, $z = 0$ (D) none of these
(E) $w = 1 - 4y - 5z$, $x = 7 + 3z$, $y = y$, $z = z$

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The water, $W$, electricity, $E$, and agriculture, $A$, sectors of an economy are related in the following way: One unit of output from $W$ requires 1/10 of a unit output from $W$, 1/10 of a unit output from $E$, and 3/10 of a unit output from $A$. One unit of output from $E$ requires 1/2 of a unit output from $W$, 1/10 of a unit output from $E$, and 1/5 of a unit output from $A$. One unit of output from $A$ requires 1/5 of a unit output from $W$, 2/5 of a unit output from $E$, and 1/5 of a unit output from $A$.

18. The coefficient matrix for this economy is:

(A) $\begin{array}{c} \\ W \\ E \\ A \end{array}\begin{array}{c} \begin{array}{ccc} W & E & A \end{array} \\ \begin{pmatrix} 1/10 & 1/10 & 3/10 \\ 1/2 & 1/10 & 1/5 \\ 1/5 & 2/5 & 1/5 \end{pmatrix} \end{array}$ (B) $\begin{array}{c} \\ W \\ E \\ A \end{array}\begin{array}{c} \begin{array}{ccc} W & E & A \end{array} \\ \begin{pmatrix} 1/10 & -1/2 & -1/5 \\ -1/10 & 1/10 & -2/5 \\ -3/10 & -1/5 & 1/5 \end{pmatrix} \end{array}$

(C) $\begin{array}{c} \\ W \\ E \\ A \end{array}\begin{array}{c} \begin{array}{ccc} W & E & A \end{array} \\ \begin{pmatrix} 1/10 & 1/2 & 1/5 \\ 1/10 & 1/10 & 2/5 \\ 3/10 & 1/5 & 1/5 \end{pmatrix} \end{array}$ (D) $\begin{array}{c} \\ W \\ E \\ A \end{array}\begin{array}{c} \begin{array}{ccc} W & E & A \end{array} \\ \begin{pmatrix} 1/10 & -1/10 & -3/10 \\ -1/2 & 1/10 & -1/5 \\ -1/5 & -2/5 & 1/5 \end{pmatrix} \end{array}$ (E) none of these

19. There is an (external) demand for 260 units of water, 140 units of electricity, and 500 units of agriculture. Let $\mathbf{A}$ be the correct matrix in question 19. Let $\mathbf{X}$ be the production matrix. The production required to satisfy the external demand is found by solving:

(A) none of these (B) $(\mathbf{A}-\mathbf{I})\mathbf{X} = \begin{bmatrix} 260 \\ 140 \\ 500 \end{bmatrix}$ (C) $(\mathbf{I}-\mathbf{A})\mathbf{X} = \begin{bmatrix} 260 \\ 140 \\ 500 \end{bmatrix}$

(D) $(\mathbf{I}+\mathbf{A})\mathbf{X} = \begin{bmatrix} 260 \\ 140 \\ 500 \end{bmatrix}$ (E) $(\mathbf{A}+\mathbf{I})\mathbf{X} = \begin{bmatrix} 260 \\ 140 \\ 500 \end{bmatrix}$

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20. The number of corner points (vertices) of the region determined by

$$\begin{cases} 5y & -2x & \leq & 10 \\ 4x & -6y & \leq & 12 \\ & y & \geq & 0 \end{cases}$$

is:

(A) 3 (B) 0 (C) 2 (D) 5 (E) 4

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The Orange Corporation produces two models of home computers: the Seville model and the Jaffa model. The factory can produce at most 650 models, combined, in a week. They write $x$ for the number of Seville models and $y$ for the number of Jaffa models produced in a week.

21. Inequalities that describe Orange's weekly production are:

(A) $\begin{cases} x + y \geq 650 \\ x \leq 0 \\ y \leq 0 \end{cases}$ (B) none of these (C) all of these (D) $\begin{cases} x + y \leq 650 \\ x \geq 0 \\ y \geq 0 \end{cases}$
(E) $\begin{cases} x + y \leq 650 \end{cases}$

22. The corner points of the region described in 21. are:

(A) $(0,0)$, $(650,0)$ (B) $(0,0)$, $(650,0)$, $(0,650)$ (C) $(0,0)$, $(0,650)$
(D) $(0,0)$, $(650,0)$, $(650,650)$, $(0,650)$ (E) $(650,0)$, $(0,650)$

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The next three questions involve the problem:

$$\text{Maximize} \qquad P = 2x_1 + x_2 - 2x_3$$

$$\text{Subject to} \quad \begin{cases} -2x_1 + x_2 + x_3 & \geq & -2 \\ x_1 - x_2 + x_3 & \leq & 4 \\ x_1 + x_2 + 2x_3 & \leq & 6 \\ x_1, x_2, x_3, x_4 & \geq & 0 \end{cases}$$

You will need to write the initial simplex table(au) for this problem, using $s_1$, $s_2$, and so on as required for the slack variables. Note that the rows are labelled by the list of basic variables and the columns are labelled by the variables and 'RHS'.

23. The first row (of numbers) of the initial table is:

(A) $\left[\begin{array}{cccccccc} -2 & 1 & 1 & 1 & 0 & 0 & 0 & -2 \end{array}\right]$ (B) $\left[\begin{array}{cccccccc} 2 & -1 & -1 & 1 & 0 & 0 & 0 & 2 \end{array}\right]$
(C) $\left[\begin{array}{cccccccc} 2 & -1 & -1 & 1 & 0 & 0 & 0 & 2 \end{array}\right]$ (D) $\left[\begin{array}{cccccccc} 2 & -1 & -1 & 1 & 0 & 0 & 0 & 2 \end{array}\right]$
(E) $\left[\begin{array}{cccccccc} 2 & -1 & -1 & -1 & 0 & 0 & 0 & 2 \end{array}\right]$

24. The pivot column is labelled by:

(A) $x_1$ (B) $x_2$ (C) $x_3$ (D) $s_1$ (E) $s_2$

25. The pivot row is labelled by:

(A) $x_1$ (B) $x_2$ (C) $s_1$ (D) $s_2$ (E) $s_3$

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**SECTION _____ NAME (PRINTED) _____**

**Student Number _____ SIGNATURE _____**

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