

EXAMINATION

SECTION _____ NAME (PRINTED) _____

Student Number _____ SIGNATURE _____

Only calculators without memory or graphics are allowed and no other electronic devices are allowed. Each multiple choice question is worth 1 mark. All work is to be shown on the attached question paper in the space provided and the answer must be selected on this answer sheet. Both the question paper and the answer sheet, both with completed identification sections, must be handed in.

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$a_{\overline{n}|r}$ is the *present* value of an ordinary annuity of \$1 per period for n periods at the interest rate of r per period.

$s_{\overline{n}|r}$ is the *future* value of an ordinary annuity of \$1 per period for n periods at the interest rate of r per period.

1. \$500 is invested at an annual rate of 8% compounded quarterly. The amount of time that the investment will take to grow to \$700 is approximately:

(A) $\frac{\ln 1.4}{\ln 1.02}$ months (B) $\ln(0.38)$ years (C) $\frac{\ln 1.4}{\ln 1.02}$ quarters (D) $\frac{\ln 1.4}{\ln 1.02}$ years (E) $\ln(0.38)$ months

2. A debt is structured so that \$2000 is due three years from now and \$3000 is due seven years from now but it may be repaid by a single payment of \$1000 now and two equal payments; the first, one year from now and the second, four years from now. The interest rate is 6% compounded annually. The equation of value at year four, enabling us to find the amount of the two equal payments is:

(A) $1000(1.06)^4 + 2x(1.06) = 2000(1.06) + 3000(1.06)^{-3}$
 (B) $1000(1.06)^4 + 2x = 2000(1.06) + 3000(1.06)^{-3}$
 (C) $1000(1.06)^4 + x(1.06)^3 + x = 2000(1.06) + 3000(1.06)^{-3}$
 (D) $1000(1.06)^4 + x(1.06)^3 + x = 2000(1.06)^{-1} + 3000(1.06)^{-3}$
 (E) $1000(1.06)^{-4} + x(1.06)^{-3} + x = 2000(1.06) + 3000(1.06)^3$

3. A machine is purchased for \$3000 down and \$250 at the end of every six months, for six years. The interest is at 8% annual rate, compounded semiannually. If the machine had been bought with an outright cash payment, the price in dollars would have been:

(A) $3000 + 250a_{\overline{12}|0.04}$ (B) $3000 - 250a_{\overline{12}|0.04}$ (C) $3000 + 250s_{\overline{12}|0.08}$
 (D) $3000 + 250a_{\overline{12}|0.08}$ (E) $3000 - 250a_{\overline{12}|0.08}$

-
4. A \$180,000 mortgage is taken out at 8% (annual rate) compounded monthly for 30 years. The finance charge (total interest paid) is:

(A) $360 \left(\frac{180,000}{s_{\overline{360}|0.08/12}} \right) - 180,000$ (B) $360 \left(\frac{180,000}{a_{\overline{360}|0.08/12}} \right)$ (C) $360 \left(\frac{180,000}{s_{\overline{360}|0.08/12}} \right)$
(D) $360 \left(\frac{180,000}{a_{\overline{360}|0.08/12}} \right) + 180,000$ (E) $360 \left(\frac{180,000}{a_{\overline{360}|0.08/12}} \right) - 180,000$

5. If $\mathbf{B} = [b_{ij}]$ is 2×2 and $b_{ij} = (-1)^{i+j}(i^2 + j^2)$ then $\mathbf{B} =$

(A) $\begin{bmatrix} -2 & 5 \\ 5 & -8 \end{bmatrix}$ (B) $\begin{bmatrix} 2 & -5 \\ -5 & 8 \end{bmatrix}$ (C) $\begin{bmatrix} 1 & -4 \\ -4 & 9 \end{bmatrix}$ (D) $\begin{bmatrix} -1 & 4 \\ 4 & -9 \end{bmatrix}$ (E) $\begin{bmatrix} 2 & 5 \\ 5 & 8 \end{bmatrix}$

6. $\begin{bmatrix} 2 & -7 \\ -6 & 4 \end{bmatrix} + \begin{bmatrix} 7 & -4 \\ -2 & 1 \end{bmatrix} + \begin{bmatrix} 2 & 7 \\ 7 & 2 \end{bmatrix} =$

(A) $\begin{bmatrix} -11 & 4 \\ 1 & -7 \end{bmatrix}$ (B) $\begin{bmatrix} -11 & 4 \\ -1 & 7 \end{bmatrix}$ (C) $\begin{bmatrix} 11 & -4 \\ 1 & -7 \end{bmatrix}$ (D) $\begin{bmatrix} 11 & -4 \\ -1 & 7 \end{bmatrix}$ (E) $\begin{bmatrix} 10 & 4 \\ 0 & 7 \end{bmatrix}$

7. If $\mathbf{A} = \begin{bmatrix} 1 & 3 & -2 \\ -2 & 1 & -1 \\ 0 & 4 & 3 \end{bmatrix}$ and $\mathbf{B} = \begin{bmatrix} 0 & -2 & 3 \\ -2 & 4 & -2 \\ 3 & 1 & -1 \end{bmatrix}$ and $\mathbf{AB} = \mathbf{C} = [c_{ij}]$ then $c_{23} =$

(A) -7 (B) 19 (C) 2 (D) 4 (E) 0

8. The matrix $\left[\begin{array}{ccc|c} 1 & 0 & 3 & -1 \\ 3 & 2 & 11 & 1 \\ 1 & 1 & 4 & 1 \\ 2 & -3 & 3 & -8 \end{array} \right]$ reduces to:

(A) none of these

(B) $\left[\begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right]$ (C) $\left[\begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right]$ (D) $\left[\begin{array}{ccc|c} 1 & 0 & 3 & -1 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right]$ (E) $\left[\begin{array}{ccc|c} 1 & 0 & 3 & -1 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right]$

9. The number of solutions of the system $\begin{cases} 3w + 5x - 4y + 2z = 0 \\ 7w - 2x + 9y + 3z = 0 \end{cases}$ is:

- (A) none (B) infinitely many (C) one (D) two (E) not many
-

10. In Leontief's Input-Output Analysis, if the *coefficient matrix* is \mathbf{A} and the *final demand* is \mathbf{D} then the *output* \mathbf{X} which will satisfy the final demand is found by solving for \mathbf{X} in:

- (A) $(\mathbf{A} - \mathbf{I})\mathbf{X} = \mathbf{D}$ (B) $(\mathbf{I} - \mathbf{D})\mathbf{X} = \mathbf{A}$ (C) $(\mathbf{A} - \mathbf{D})\mathbf{X} = \mathbf{I}$ (D) $\mathbf{X} = \mathbf{AD}$
 (E) $(\mathbf{I} - \mathbf{A})\mathbf{X} = \mathbf{D}$
-

Questions 11., 12., 13., and 14. involve the problem:

$$\begin{array}{l} \text{Maximize} \quad P = 2x_1 + x_2 - 2x_3 \\ \text{Subject to} \quad \begin{cases} -2x_1 + x_2 + x_3 \geq -2 \\ x_1 - x_2 + x_3 \leq 4 \\ x_1 + x_2 + 2x_3 \leq 6 \\ x_1, x_2, x_3 \geq 0 \end{cases} \end{array}$$

for which it can be shown that the *second* simplex table(au) is:

B	x_1	x_2	x_3	s_1	s_2	s_3	P	R
x_1	1	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	0	0	0	1
s_2	0	$-\frac{1}{2}$	$\frac{3}{2}$	$-\frac{1}{2}$	1	0	0	3
s_3	0	$\frac{3}{2}$	$\frac{5}{2}$	$-\frac{1}{2}$	0	1	0	5
P	0	-2	1	1	0	0	1	2

11. This table(au) tells us that:

- (A) Maximum is $P = 2$ at $(1, 0, 0)$ (B) Maximum is $P = 2$ at $(1, 3, 5)$ (C) $P = 2$ at $(1, 0, 0)$
 (D) $P = -2$ at $(1, 0, 0)$ (E) $P = 2$ at $(1, 3, 5)$

12. The pivot column in the table(au) shown is the one labelled by:

- (A) x_2 (B) x_3 (C) x_1 (D) P (E) s_1

13. The pivot row in the table(au) shown is the one labelled by:

- (A) s_2 (B) P (C) x_1 (D) s_3 (E) B

14. The solution to this maximization problem is:

- (A) $P = 2$ at $(\frac{8}{3}, \frac{10}{3}, 0)$ (B) $P = 2$ at $(\frac{8}{3}, \frac{14}{3}, \frac{10}{3})$ (C) $P = \frac{26}{3}$ at $(\frac{8}{3}, \frac{14}{3}, \frac{10}{3})$
 (D) $P = \frac{26}{3}$ at $(\frac{8}{3}, \frac{10}{3}, 0)$ (E) none of these
-

15. A person lives in city A and commutes by car to city B, which is connected to city A by 5 roads. The number of possible round trip routes which do not use the same road in both directions is:

- (A) 16 (B) 20 (C) 25 (D) 9 (E) 8
-

16. A personnel director must hire 6 people: 4 for the assembly line and 2 for the shipping dock. There are 10 applicants, all of whom are equally qualified for either type of job. The number of ways the personnel director can fill the positions is:

- (A) $10 \cdot 9$ (B) $40 \cdot 18$ (C) $\frac{10!}{4! \cdot 2!}$ (D) $\frac{10!}{4!}$ (E) $\frac{10!}{4! \cdot 2! \cdot 4!}$
-

17. From a hat containing 9 distinguishable rabbits, 4 rabbits are pulled successively without replacement. The number of possible outcomes for this experiment is:

- (A) $9 \cdot 8 \cdot 7 \cdot 6$ (B) 9^4 (C) $9 \cdot 4$ (D) $9 + 8 + 7 + 6$ (E) $\frac{9!}{4! \cdot 5!}$
-

Questions 18. and 19. concern the following situation: Out of 3000 tires in a warehouse, 2000 are domestic and the rest are imported. 40% of the domestic tires are whitewalls and 10% of the imported tires are whitewalls. A tire is selected at random and is observed to be a whitewall.

18. The probability that the tire is imported is:

- (A) none of these (B) $\frac{8}{9}$ (C) $\frac{1}{10}$ (D) $\frac{1}{9}$ (E) $\frac{1}{3}$

19. The probability that the tire is domestic is:

- (A) $\frac{2}{3}$ (B) none of these (C) $\frac{1}{9}$ (D) $\frac{2}{5}$ (E) $\frac{4}{15}$
-

20. You and a friend agree to play the following game: You will toss three fair coins. If three heads or three tails turn up, your friend is to pay you \$10. If any other outcome occurs, you are to pay your friend \$6. Your expected winnings in this game, in dollars, is:

- (A) 2 (B) 10 (C) 8 (D) -6 (E) -2
-

21. From a deck of 52 playing cards, 3 cards are randomly selected with replacement. The probability of selecting exactly 2 aces is:

- (A) ${}_3P_2 \left(\frac{1}{13}\right)^2 \left(\frac{12}{13}\right)$ (B) $\left(\frac{1}{13}\right)^2$ (C) ${}_3C_2 \left(\frac{1}{13}\right)^2 \left(\frac{12}{13}\right)$ (D) ${}_3C_2 \left(\frac{1}{13}\right)^2$ (E) none of these
-

22. If interest is compounded continuously then the annual rate r at which a principal will triple in 20 years is:

- (A) none of these (B) $\frac{3}{20e}$ (C) $\frac{\ln 3}{20}$ (D) $\frac{\ln 3}{\ln 20}$ (E) $\frac{3e}{20}$
-

23. The slope of the curve $y = f(x) = 2 - 3x^2$ at the point $(1, -1)$ is:

- (A) $\lim_{h \rightarrow 0} \frac{(2 - 3(1+h)^2) - (2 - 3(1)^2)}{h}$ (B) none of these (C) $\lim_{h \rightarrow 0} \frac{(2 - 3(1+h)^2) + (2 - 3(1)^2)}{h}$
(D) $\lim_{h \rightarrow 0} \frac{(2 - 3(1)^2 + h) - (2 - 3(1)^2)}{h}$ (E) $\lim_{h \rightarrow 0} \frac{(2 - 3(1+h)^2)}{h}$
-

24. If $f(t) = -13t^2 + 14t + 1$ then $f'(t) =$

- (A) $-26t$ (B) $-26t + 14t + 1$ (C) $-12t$ (D) $-26t + 14$ (E) $-26t^2 + 14$
-

25. If cost c is given as a function of quantity q by

$$c = 0.04q^3 - 0.5q^2 + 4.4q + 7500$$

then the marginal cost when $q = 10$ is:

- (A) 0.12 (B) 6.4 (C) 4.4 (D) -1 (E) 4.52
-

26. If $\phi(x) = (3 - 5x + 2x^2)(2 + x - 4x^2)$ then $\phi'(x) =$

- (A) $(-5 + 4x)(2 + x - 4x^2) + (3 - 5x + 2x^2)(1 - 8x)$ (B) $(-5 + 4x)(1 - 8x)$
(C) $(-5 + 4x)(2 + x - 4x^2) - (3 - 5x + 2x^2)(1 - 8x)$
(D) $\frac{(-5 + 4x)(2 + x - 4x^2) - (3 - 5x + 2x^2)(1 - 8x)}{x^2}$ (E) none of these
-

27. If $y = (x^2 - 4)^4$ then $\frac{dy}{dx} =$

- (A) $4(x^2 - 4)^3(2x)$ (B) $4(2x)^3$ (C) $4(x^2 - 4)^3 + (2x)$ (D) $4(2x)^3(2x)$ (E) $4(2x)^3 + (2x)$
-

28. If $y = (3x + 2)^5(4x - 5)^2$ then $\frac{dy}{dx} =$

- (A) none of these (B) $5(3)^4(4x - 5)^2 + (3x + 2)^5 2(4)$ (C) $5(3x + 2)^4(3)2(4x - 5)(4)$
(D) $5(3x + 2)^4(3)(4x - 5)^2 - (3x + 2)^5 2(4x - 5)(4)$ (E) $5(3x + 2)^4(3)(4x - 5)^2 + (3x + 2)^5 2(4x - 5)(4)$
-

In problems 29. and 30. we are given a manufacturer's average cost \bar{c} as a function of quantity q by:

$$\bar{c} = \frac{350}{\ln(q + 2)}$$

29. The manufacturer's cost c as a function of q is:

- (A) $\frac{350}{\ln(q + 2)}$ (B) $\frac{350q}{\ln(q + 2)q}$ (C) $\frac{350}{\ln(q + 2)q}$ (D) $\frac{350}{\ln(q + 2)} + q$ (E) $\frac{350q}{\ln(q + 2)}$

30. The manufacturer's marginal cost c' as a function of q is:

- (A) $350 \frac{\ln(q + 2)(1) + q \left(\frac{1}{q + 2} \right)}{(\ln(q + 2))^2}$ (B) $350 \frac{\ln(q + 2)(1) - q \left(\frac{1}{q + 2} \right)}{(\ln(q + 2))^2}$
(C) $350 \frac{q \left(\frac{1}{q + 2} \right) - \ln(q + 2)(1)}{(\ln(q + 2))^2}$ (D) $350 \frac{\ln(q + 2)(1) - q \left(\frac{-1}{(q + 2)^2} \right)}{(\ln(q + 2))^2}$
(E) $350 \frac{\ln(q + 2)(1) - q \left(\frac{1}{q + 2} \right)}{(\ln(q + 2))^4}$
-

Questions 31. and 32. concern the function $f(x) = 10^{-x} + \ln(8 + x) + 0.01e^{x-2}$.

31. The *relative rate of change of $f(x)$* is:

- (A) $\frac{-(\ln 10)10^{-x} + \frac{1}{8+x} + 0.01e^{x-2}}{x}$ (B) $\ln\left(-(\ln 10)10^{-x} + \frac{1}{8+x} + 0.01e^{x-2}\right)$
(C) $\ln(10^{-x} + \ln(8+x) + 0.01e^{x-2})$ (D) $\frac{-(\ln 10)10^{-x} + \frac{1}{8+x} + 0.01e^{x-2}}{10^{-x} + \ln(8+x) + 0.01e^{x-2}}$ (E) none of these

32. When $x = 2$ the relative rate of change of $f(x)$ is:

- (A) $\frac{-(\ln 10)10^{-2} + \frac{1}{10} + 0.01}{10^{-2} + \ln(10) + 0.01}$ (B) $\ln\left(-(\ln 10)10^{-2} + \frac{1}{10} + 0.01\right)$ (C) $\ln(10^{-2} + \ln(10) + 0.01)$
(D) $\frac{-(\ln 10)10^{-2} + \frac{1}{10} + 0.01}{2}$ (E) $\frac{-(\ln 10)10^{-2}}{2}$

Questions 33., 34., and 35. concern the demand equation $q = \sqrt{2500 - p^2}$

33. The elasticity of demand is given by:

- (A) $\frac{-p^2}{2500 + p^2}$ (B) $\frac{p^2}{2500 - p^2}$ (C) $\frac{2500 - p^2}{-p^2}$ (D) $\frac{-p^2}{2500 - p^2}$ (E) $\frac{p^2}{2500 + p^2}$

34. If price is 30 then the elasticity of demand is:

- (A) $-\frac{9}{16}$ (B) $-\frac{9}{34}$ (C) $\frac{9}{16}$ (D) $-\frac{16}{9}$ (E) $-\frac{34}{9}$

35. If the price of 30 is decreased by 10% then demand will:

- (A) increase by 10% (B) decrease by 10% (C) increase by $\frac{45}{16}\%$
(D) increase by $\frac{90}{16}\%$ (E) decrease by $\frac{90}{16}\%$

36. If $y = \frac{x}{e^x}$ then $\frac{d^2y}{dx^2} =$

- (A) $\frac{-1}{(e^x)^2}$ (B) $\frac{x-2}{e^x}$ (C) $\frac{1-x}{e^x}$ (D) $\frac{1-x}{(e^x)^2}$ (E) $\frac{x-2}{(e^x)^2}$
-

Questions 37. and 38. concern the function given by $f(x) = \frac{1}{4}x^4 - \frac{1}{2}x^2 + 3$ on the closed interval $[0, 3]$.

37. The absolute maximum value of f on $[0, 3]$ is:

(A) $f(0) = 3$ (B) $f(1) = \frac{11}{4}$ (C) $f(1) = \frac{75}{4}$ (D) $f(0) = \frac{75}{4}$ (E) $f(3) = \frac{75}{4}$

38. The absolute minimum value of f on $[0, 3]$ is:

(A) $f(3) = \frac{75}{4}$ (B) $f(3) = \frac{11}{4}$ (C) $f(1) = \frac{11}{4}$ (D) $f(0) = \frac{11}{4}$ (E) $f(0) = 3$

Questions 39., 40., 41., and 42. are concerned with the function: $y = x^3 - 25x^2$.

39. The function is increasing and decreasing as follows:

- (A) increasing on $(-\infty, 0)$, decreasing on $(0, 25)$, and increasing on $(25, \infty)$
(B) decreasing on $(-\infty, 0)$, increasing on $(0, \frac{50}{3})$, and decreasing on $(\frac{50}{3}, \infty)$
(C) increasing on $(-\infty, 0)$, decreasing on $(0, \frac{50}{3})$, and increasing on $(\frac{50}{3}, \infty)$
(D) decreasing on $(-\infty, 0)$, increasing on $(0, 25)$, and decreasing on $(25, \infty)$ (E) none of these

40. The relative extrema are as follows:

- (A) relative minimum at $x = 0$; relative maximum at $x = 25$
(B) relative minimum at $x = 0$; relative maximum at $x = \frac{50}{3}$
(C) relative maximum at $x = 0$; relative minimum at $x = \frac{50}{3}$
(D) relative maximum at $x = 0$; relative minimum at $x = 25$ (E) none of these

41. The concavity of the function is as follows:

- (A) none of these (B) concave up on $(-\infty, \frac{25}{3})$; concave down on $(\frac{25}{3}, \infty)$
(C) concave down on $(-\infty, 0)$; concave up on $(0, \frac{50}{3})$; concave down on $(\frac{50}{3}, \infty)$
(D) concave up on $(-\infty, 0)$; concave down on $(0, \frac{50}{3})$; concave up on $(\frac{50}{3}, \infty)$
(E) concave down on $(-\infty, \frac{25}{3})$; concave up on $(\frac{25}{3}, \infty)$

42. The function has point(s) of inflection at:

- (A) $x = 0$ and $x = \frac{25}{3}$ (B) $x = \frac{25}{3}$ (C) $x = 0$, $x = \frac{25}{3}$, and $x = \frac{50}{3}$
(D) $x = \frac{25}{3}$ and $x = \frac{50}{3}$ (E) $x = 0$

43. The *horizontal* asymptotes of the curve $y = \frac{x+1}{x}$ are:

- (A) $x = 1$ (B) $y = 1$ (C) $x = 0$ (D) $y = 0$ (E) $y = 2$
-

Questions 44., 45., and 46. concern the following problem: A monopolist's cost per unit is \$3 and the demand equation for his product is: $p = \frac{10}{\sqrt{q}}$, where p is in dollars and q is in units of product. He would like to set his price so as to enjoy the greatest profit.

44. Writing P for profit and expressing it as a function of demand q , the monopolist needs to maximize:

- (A) $P = \left(\frac{10}{\sqrt{q}}\right)q$, for $q > 0$ (B) $P = \left(\frac{10}{\sqrt{q}}\right) - 3q$, for $q > 0$ (C) $P = \left(\frac{10}{\sqrt{q}}\right)q - 3$
(D) $P = \left(\frac{10}{\sqrt{q}}\right)q - 3q$, for $q > 0$ (E) none of these

45. P attains a maximum when $q =$

- (A) $\frac{5}{3}$ (B) $\frac{25}{9}$ (C) $\frac{10}{3}$ (D) $\frac{10}{9}$ (E) none of these

46. The price (in dollars) which maximizes profit is $p =$

- (A) $\frac{90}{25}$ (B) 3 (C) 9 (D) 10 (E) 6

Questions 47. and 48. ask for the partial derivatives of $g(x, y) = (x + 1)^2 + (y - 3)^3 + 5xy^3 - 2$.

47. $g_x(x, y) =$

- (A) $3(y - 3)^2 + 15xy^2$ (B) $2(x + 1)$ (C) $2(x + 1) + 5y^3$ (D) $15x$ (E) $5y^3$

48. $g_y(x, y) =$

- (A) $5y^3$ (B) $3(y - 3)^2 + 15y^2$ (C) $3(y - 3)^2 + 15xy^2$ (D) $3(y - 3)^2$ (E) $2(x + 1) + 5y^3$

Questions 49. and 50. concern products A and B, with corresponding prices p_A and p_B and corresponding demands, each depending on both prices, as follows: $q_A = 20 - p_A - 2p_B$ and $q_B = 50 - 2p_A - 3p_B$.

49. If the price of A increases then:

- (A) demand for A decreases for any fixed price of B
(B) demand for A decreases for low fixed prices of B
(C) demand for A decreases for high fixed prices of B
(D) demand for A increases for low fixed prices of B
(E) demand for A increases for any fixed price of B

50. The products A and B can be classified as:

- (A) competitive (B) neither competitive nor complementary (C) none of these
(D) complementary (E) all of these
-