Examples for Section 3.1

1. Tossing a coin, define a random variable for this experiment.

2. Tossing a dice, write out the sample space, define a random variable.

3. Two gas stations, each with 6 pumps. If the numbers of pumps in use at a purticular time of the day are interested, write out the sample space. Define random variables as below:
   X: The total number of pumps in use at the two stations.
   Y: The difference between the no. of pumps in use in station 1 and that of station 2.
   Z: The maximum of the numbers of pumps in use at two stations.

4. For sample space S={H, TH, TTH, ...}, define a random variable.

Examples for Section 3.2

Example 1 :

Tossing the coin twice. Head fell *x* times. *x* is a *random variable*.

x	Events	Probability
0	(T,T)	1/4
1	(H,T), (T,H)	1/2
2	(H,H)	1/4

Example 2 (Ex. 3.9)

Consider a group of 5 potential blood donors, A, B,C, D, and E, of whom only A and B have type O+ blood. Five blood samples, one from each individual, will be typed in random order until an O+ individual is identified. Let the R.V. Y= the number of typings necessary to identify an O+ individual. Write out the pmf of Y.

Example 3

For what values of the constant c do the function $f(x)=c2^{-x}$ defines a pmf on the positive integers 1, 2, 3,...?
For the R.V. having the above pmf, find P(X>1) and P(X is even).

Example 4 (Ex. 3.11)

Calculate the cdf for the above example 1, and draw the plot for the cdf.

Example 5

For the above example 2, write out the cdf F(x). From the cdf, calculate $P(X>1)$ and $P(2 \le X \le 5)$.

Could you get pmf from the cdf F(x)?

Examples for Section 3.3

Example 1:

The probability to win \$1,000,000 in sweepstakes is 1:1000,000. Is it reasonable to buy a \$5 ticket?

Answer:

If I play many times, I win \$1,000,000 × 0.000,000,1=\$1 in each game. The house has \$4!

Example 2. Let X be bernoulli random variable with pmf $p(x)=1-p$ if $X=0$ and p if $X=1$. Calculate E(X).

Example 3. Calculate the mean if pmf is given as $p(x)=(1/2)^x$, x=1,2,....

Example 4. Let X be the number of cylinders in the engine of the next car to be tuned up at a certain facility, the cost of a tune-up is $h(X)=20+3X+0.5x^2$, denote h(X) as random variabe Y. If the pmf of X is given as

x	4	6	8
p(x)	0.5	0.3	0.2

What is the pmf of Y? Calculate the mean of Y variable.

Example 5. For the above example, calculate the variance and the standard deviation of X and Y.

Examples for Section 3.4

Example 1. Toss the same coin 4 times, suppose for each tossing, P(H)=p. Denote random variable X as the number of heads in 4 times tossing.

1. What's the all possible values of X?
2. What the pmf of X?
3. Mean of X?
4. Variance of X?

Example 2 . Customers are a gas station pay with a credit card (A), debit card (B), or cash (C). Assume that successive customers make independent choices with P(A)=0.5, P(B)=0.2 and P(C) =0.3.

1. Among the nect 20 customers, what's the probability that 10 will pay by credit card?
2. What are the mean and variance of the number of customers who pay with a debit card.
3. What's the probability that the number of customers who pay by cash is within 10 to 15?

Examples for Section 3.5

Example 1 . Suppose there are 25 copies of text books in library, of which 7 are 4^{th} edition, 18 are 5^{th} edition. The instructor wants to request that 5 copies be put on reserve, if copies of books being put on reserve are selected randomly, and denote random variable X=no. Of copied put on reserved are 4^{th} edition.

1. What is the pmf of X?
2. If the instructor requests to reserve 8 copies, what will be pmf of X again?

Example 2. A couple wishes to have exactly two female children in their family, they decide to have children until this condition is fulfilled. Supoose that P(the new birth is famale)=0.4.

1. What's the probability that the family has x male children?
2. What's the probability that the family has children?
3. What's the probability that the family has at most 4 children?
4. How many male children would you expect this family to have?
5. How many children would you expect this family to have?

Examples for Section 3.6

Poisson distribution

Problem:

There are on average $\lambda = 3$ major airplane crashes per year in the USA. What is the probability to have exactly x=5 of them this year?

Assumption:

Catastrophes are independent of each other and can happen any time.

Notation:

$P(x,t)$--probability to have x crashes over time t. T--one year.

Small time Δt :

Probability of a catastrophe $\lambda \Delta t / T$.

Equation for $P(0, t + \Delta t)$:

No crashes at the time t, no new crashes in the interval Δt . Independent events:

$$P(0, t + \Delta t) = P(0, t)\left(1 - \frac{\lambda \Delta t}{T}\right)$$

Therefore

$$\frac{dP(0,t)}{dt} = -\frac{\lambda}{T}P(0,t)$$

$$P(0,0) = 1$$

Solution for $P(0,t)$:

$$P(0,t) = e^{-\lambda t/T}$$

Equation for $P(1, t + \Delta t)$:

Two possibilities:

1. One catastrophe during the time t, no new ones during Δt .
2. No catastrophes during t, one catastrophe during Δt .

These possibilities are *mutually exclusive*

$$P(1, t + \Delta t) = P(1, t)\left(1 - \frac{\lambda \Delta t}{T}\right) + P(0, t)\frac{\lambda \Delta t}{T}$$

Therefore

$$\frac{dP(1,t)}{dt} = -\frac{\lambda}{T}P(1,t) + \frac{\lambda}{T}e^{\lambda t/T}$$

$$P(1,0) = 1$$

Solution for *P*(1,*t*):

$$P(1,t) = \frac{\lambda t}{T}e^{-\lambda t/T}$$

General case:

$$P(x,t) = \frac{\lambda^x t^x}{T^x x!}e^{-\lambda t/T}$$

Poisson distribution:

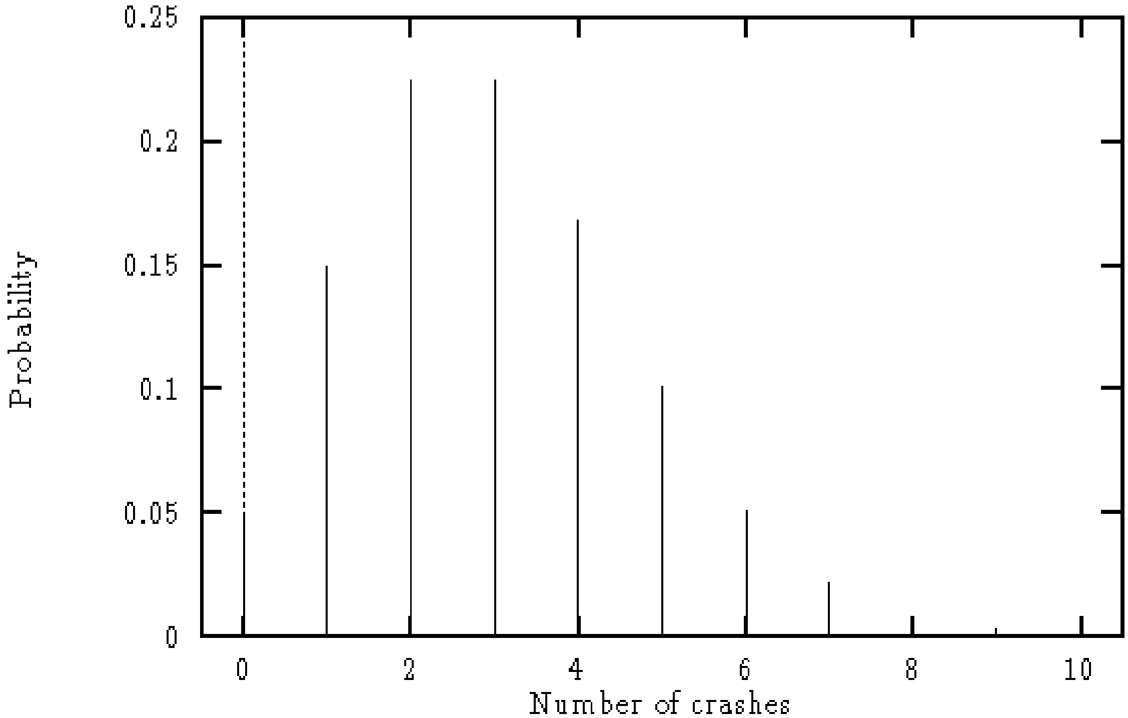
t=T (one year!)

$$P(x,T) = \frac{\lambda^x}{x!}e^{-\lambda}$$

Air crashes:

$\lambda = 3$, x=5,

$$P = \frac{3^5}{5!}e^{-3} = 0.10$$



Example 2. The number of requests for assistance received by a towing service is a Poisson process with with rate α=4 per hour.

1. What's the probability that exactly 10 requests are received during a particular 2-hour period?
2. If the operators of the towing service take a 30-minute break for lunch, what's the probability that they do not miss any calls for assistance?
3. How many calls would you expect during their break?

Example 3. A publisher tries to ensure their books are free of typos. The probability of any given page containing at least one typo error is 0.005, assumingtypo errors are independent from page to page, what's the probability that one of its 400-pages novel will contain exactly one page with errors? At most three pages with errors?