Chapter4 : Examples

Section 4.1

- Prove that

$$f(x) = \begin{cases} 1/10, & 0 \leq x \leq 10 \\ 0, & otherwise \end{cases} \tag{1}$$

  is a probability density function; Calculate $P(2 \leq X \leq 5)$.

- A college prof. always finishes his lectures within 2 minutes after the bell rings. Let X denote the random variable which is the time that elapses between the bell and the end of the lecture. Suppose the pdf of X is

$$f(x) = \begin{cases} kx^2, & 0 \leq x \leq 2 \\ 0, & otherwise \end{cases} \tag{2}$$

  - Find the value of k.
  - $P(X \leq 1)$
  - $P(1 \leq X \leq 1.5)$
  - $P(x > 1.5)$

Section 4.2

- $X \sim Unif[A, B]$, calculate the cdf of X.

- X follows exponential distribution if

$$f(x; \lambda) = \begin{cases} \lambda e^{-\lambda x}, & x \geq 0 \\ 0, & otherwise \end{cases}$$

  Where $\lambda > 0$. Write out the cdf of X and calculate $P(X > 1)$

- If cdf

$$F(x) = \begin{cases} 1 - exp(-\lambda x), & x \geq 0 \\ 0, & otherwise \end{cases}$$

  calculate the pdf of X.

- If X follows Unif[A,B], write out the cdf of X, derive the pdf of X from its cdf.
- For the above exponential distribution,

$$f(x;\lambda) = \begin{cases} \lambda e^{-\lambda x}, & x \geq 0 \\ 0, & otherwise \end{cases}$$

  calculate the mean E(X) and Var(X).
- Given pdf of X:

$$f(x) = \begin{cases} 3/2(1-x^2), & 0 \leq x \leq 1 \\ 0, & otherwise \end{cases}$$

  Calculate the 30th percentile η.

Section 4.3

- If $X \sim N(3, 2.5)$, calculate $P(2 < X < 5.5)$.
- Calculate the 95th percentile of N(3,2).
- Let $X \sim Bin(25, 0.6)$, use Normal approximation to calculate $P(X \leq 15)$ and $P(X \geq 20)$.