Stat2060 problem set

This problem set is only intended to help you to get familiar with the material you learned in this course. Most of the questions in this problem set are from the exams before. None of the questions in our exam will be from this problem set. The questions in this set have been arranged in an arbitrary manner, so that it may help you to improve your ability to recognize the type of questions in the exam. You are suggested to use these questions only as supplementary help for your review. Solutions of these questions will not be given.

1. X is a continuous random variable with probability density function

$$f(x) = \begin{cases} 1.5x^2 & -1 \le x \le 1 \\ 0 & otherwise \end{cases}$$

   (a) Verify that $f(x)$ is a valid p.d.f.
   (b) Find $\mathbf{E}[X^m]$ for m an even positive integer.
   (c) Find $\mathbf{E}[X^m]$ for m an odd positive integer.
   (d) Find the standard deviation of X.
   (e) Find the 75'th percentile of X.

2. A company has 40 employees, 20 of which are women. Among these 20 women, 4 are minorities. A committee of 4 people is selected at random, with all committees begin equally likely.

   (a) What is the probability that more than half of the committee will be women?
   (b) Given that there is exactly one minority woman on the committee, what is the probability that there are no men?
   (c) Determine the probability of the event that there is at least one non-minority woman, at least one man and at least one minority woman on the committee. Note that the probability of a single event is being asked for.

3. Prior work suggests that the true mean aerobic capacity of Peruvian highland natives is 43.5. Measurements of aerobic capacities were recorded for a group of 21 Peruvian highland natives. The sample mean and standard deviation were 46.3 and 5.0, respectively. Does the sample provide evidence that the population mean aerobic capacity is greater than 43.5?

   (a) Calculate the observed test statistic?
   (b) Calculate the p-value to as much accuracy as possible.

(c) What is your conclusion at level $\alpha = .05$?

4. Suppose that X is a continuous random variable with a uniform distribution on the interval [0,1]. Let A_1 be the event $\{.1 \le X \le .25\}$, A_2 be the event $\{.15 \le X \le .35\}$ and A_3 be the event $\{.75 \le X \le .8\}$.

   (a) Find $\mathbf{P}(A_1 \cup A_2 \cup A_3)$.

   (b) Find $\mathbf{P}(A_1 \cap A_2')$.

5. It is known that 20% of all female high school students smoke at least one cigarette per day. If 100 female high school students are randomly sampled,

   (a) What are the mean and variance of the number in the sample who smoke at least one cigarette per day?

   (b) What is the probability that 15 or fewer of the students sampled smoke at least one cigarette per day?

6. The 95% confidence interval for μ, the true mean iron concentration in drinking water from a certain area is (6.2,9.8) parts per million (ppm). If you wished to test the null hypothesis $\mu = 9.0$ ppm against the alternate hypothesis $\mu \neq 9.0$ ppm, would you reject the null hypothesis at $\alpha = .10$? Explain.

7. The probability mass function for a random variable X is given by the following table

$p(x)$	1/4	1/2	1/8	1/8
x	-1	2	5	6

   (a) Find $E[e^X]$.

   (b) Find the cumulative distribution function of X.

8. The probability that any given calculator produced by a manufacturer is defective is 0.2.

   (a) What is the probability that out of 10 calculators produced, at least 1 will be found to be defective?

   (b) If calculators that are defective must be destroyed at a cost of $50 and calculators sold give a profit of $20 to the producer, what is the expected net profit for the producer in selling 30 calculators?

   (c) What is the probability that exactly 10 good calculators will be produced before the first defective calculator is produced?

9. Let A, B and C denote events.

   (a) Suppose that $P(A) = 0.7$, $P(B) = 0.5$ and $P(A \cap B) = 0.25$. Find $P(A' \cup B')$.

(b) Suppose that in addition to the probabilities in part (a) we have that $P(C) = 0.60$, $P(A \cap B \cap C) = 0.2$, $P(A \cap C) = 0.25$ and $P(B \cap C) = 0.4$. Find $P(A \cap B^C \cap C)$.

10. A chest contains three drawers. The first contains 2 gold coins, the second contains one gold and one silver coin, and the third contains two silver coins. A drawer is chosen randomly, and then one of the two coins in the chosen drawer is randomly selected.

    (a) What is the probability that the selected coin is gold?

    (b) Given that the selected coin is gold, what is the probability that the other coin in the same drawer is gold?

11. In a game of roulette a ball is spun and lands on one of 18 red, 18 black or 2 green slots. If the game is fair the ball should be equally likely to fall in any one of the 38 slots. Suppose that the ball is rolled 500 times and lands on red 256 times. Based upon this data, test whether the game is fair?

    (a) State the null and alternative hypotheses, find the test statistic and report a p-value.

    (b) Construct a 99% confidence interval for the true probability of landing on red.

12. The heights of tomato plants have a mean of 25.65 inches. Ten tomato plants are treated with Gro-food. The average height of the 10 plants is 26.85 inches and $s^2 = 2.92$. Suppose that the heights of the treated tomato plants are normally distributed with a mean μ and variance σ^2.

    (a) Determine the p-value for the test of $H_0 : \mu = 25.65$ against the alternative hypothesis $H_A : \mu \neq 25.65$, and report the conclusion of a test at level of significance $\alpha = 0.05$.

    (b) Construct a 95% confidence interval for the mean height of plants treated with Gro-food.

13. Suppose that X is a normal random variable with mean 10 and variance 9.

    (a) Find the 50'th percentile of X.

    (b) Find the 75'th percentile of X.

14. The heights of female students at Dalhousie can be viewed as being normally distributed with a mean of 64 inches and a standard deviation of 2.5 inches. The heights of male students are normally distributed with mean 68 and standard deviation 3. One male and one female student are randomly selected. What is the probability that the absolute difference in their heights is less than 4 inches?

15. The amount of beer (in fluid ounces) which an assembly line machine dispenses into a bottle of beer is known to be normally distributed with mean μ and variance σ^2. The machine fills bottles independently of one another.

    (a) Suppose that the variance σ^2 is known to be equal to .04, and a technician wishes to tune the machine by setting the mean μ so that the probability that a case of beer (12 bottles) receives more than 140 fluid ounces is .95. Find μ.

    (b) Suppose that both the mean μ and the variance σ^2 are unknown, and the technician wishes to set them so that the probability that a case of beer (12 bottles) receives more than 140 fluid ounces is .95 and the probability that a case of beer receives less than 146 fluid ounces is .99. Find μ and σ.

16. Over the long run, a manufacturing process produces 1% defective items.

    (a) What is the probability of getting exactly 5 defective items in a sample of 100 items?

    (b) what is the probability of getting more than 3 defective items in a sample of 200 items?

17. Customers arrive at a cashier at a rate of 2 per minute.

    (a) What is the probability that there is a rush of more than 5 customers in a given minute?

    (b) What is the probability that the cashier stands idle in a given minute?

18. An inspector working for a manufacturing company has a 99% cnance of correctly identifying defective items and a 0.5% chance of incorrectly classfying a good item as defective. the company has evidence that its line produces 0.9% of noncomforming items.

    (a) What is the probability that an item selected for inspection is classified as defective?

    (b) If an item selected at random is classified as nondefective, what is the probability that it is indeed good?