Assignment 11:

Chapter 8

Questions 12, 18, 20, 26, 30, 32, 38, 42, 46, 48, 52

12.

a. Let $\mu =$ true average braking distance for the new design at 40 mph. The hypotheses are $H_o : \mu = 120$ vs $H_a : \mu < 120$.

b. R_2 should be used, since support for H_a is provided only by an $\bar{x}$ value substantially smaller than 120. ($E(\bar{x}) = 120$ when H_o is true and , 120 when H_a is true).

c. $\sigma_{\bar{x}} = \dfrac{\sigma}{\sqrt{n}} = \dfrac{10}{6} = 1.6667$, so $\alpha = P(\bar{x} \geq 115.20$ when $\mu = 120)$ = $P\left(z \leq \dfrac{115.20 - 120}{1.6667}\right) = P(z \leq -2.88) = .002$. To obtain $\alpha = .001$, replace 115.20 by $c = 120 - 3.08(1.6667) = 114.87$, so that $P(\bar{x} \leq 114.87$ when $\mu = 120)$ = $P(z \leq -3.08) = .001$.

d. $\beta(115) = P(\bar{x} > 115.2$ when $\mu = 115) = P(z > .12) = .4522$

e. $\alpha = P(z \leq -2.33) = .01$, because when H_o is true Z has a standard normal distribution ($\bar{X}$ has been standardized using 120). Similarly $P(z \leq -2.88) = .002$, so this second rejection region is equivalent to R_2.

18.

a. $\dfrac{72.3 - 75}{1.8} = -1.5$ so 72.3 is 1.5 SD's (of $\bar{x}$) below 75.

b. H_o is rejected if $z \leq -2.33$; since $z = -1.5$ is not ≤ -2.33, don't reject H_o.

c. $\alpha =$ area under standard normal curve below –2.88 $= \Phi(-2.88) = .0020$

d. $\Phi\left(-2.88 + \dfrac{75 - 70}{9/5}\right) = \Phi(-.1) = .4602$ so $\beta(70) = .5398$

e. $n = \left[\dfrac{9(2.88 + 2.33)}{75 - 70}\right]^2 = 87.95$, so use n = 88

f. $\alpha(76) = P(Z < -2.33$ when $\mu = 76) = P(\bar{X} < 72.9$ when $\mu = 76)$

$$= \Phi\left(\frac{72.9-76}{.9}\right) = \Phi(-3.44) = .0003$$

20. With H_o: $\mu = 750$, and H_a: $\mu < 750$ and a significance level of .05, we reject H_o if z < -1.645; z = -2.14 < -1.645, so we reject the null hypothesis and do not continue with the purchase. At a significance level of .01, we reject H_o if z < -2.33; z = -2.14 > -2.33, so we don't reject the null hypothesis and thus continue with the purchase.

26. Reject H_o if $z \geq 1.645$; $\frac{s}{\sqrt{n}} = .7155$, so $z = \frac{52.7-50}{.7155} = 3.77$. Since 3.77 is ≥ 1.645, reject H_o at level .05 and conclude that true average penetration exceeds 50 mils.

30. n = 115, $\bar{x} = 11.3$, $s = 6.43$

1. Parameter of Interest: $\mu =$ true average dietary intake of zinc among males aged 65 – 74 years.
2. Null Hypothesis: H_o: $\mu = 15$
3. Alternative Hypothesis: H_a: $\mu < 15$
4. $z = \frac{\bar{x}-\mu_o}{s/\sqrt{n}} = \frac{\bar{x}-15}{s/\sqrt{n}}$
5. Rejection Region: No value of α was given, so select a reasonable level of significance, such as α= .05. $z \leq z_\alpha$ or $z \leq -1.645$
6. $z = \frac{11.3-\mu_o}{6.43/\sqrt{115}} = -6.17$
7. –6.17 < -1.645, so reject H_o. The data does support the claim that average daily intake of zinc for males aged 65 - 74 years falls below the recommended daily allowance of 15 mg/day.

32. n = 12, $\bar{x} = 98.375$, $s = 6.1095$

a.

1 Parameter of Interest: $\mu =$ true average reading of this type of radon detector when exposed to 100 pCi/L of radon.

2 Null Hypothesis: H_o: $\mu = 100$

3 Alternative Hypothesis: H_a: $\mu \neq 100$

4 $$t = \frac{\bar{x} - \mu_o}{s/\sqrt{n}} = \frac{\bar{x} - 100}{s/\sqrt{n}}$$

5 $t \le -2.201$ or $t \ge 2.201$

6 $$t = \frac{98.375 - 100}{6.1095/\sqrt{12}} = -.9213$$

7 Fail to reject H_o. The data does not indicate that these readings differ significantly from 100.

b. $\sigma = 7.5$, $\beta = 0.10$. From table A.17, df ≈ 29, thus n ≈30.

38.

a. We wish to test H_o: p = .02 vs H_a: p < .02; only if H_o can be rejected will the inventory be postponed. The lower-tailed test rejects H_o if $z \le -1.645$. With $\hat{p} = \frac{15}{1000} = .015$, z = -1.01, which is not ≤ -1.645. Thus, H_o cannot be rejected, so the inventory should be carried out.

b. $$\beta(.01) = 1 - \Phi\left[\frac{.02 - .01 - 1.645\sqrt{.02(.98)/1000}}{\sqrt{.01(.99)/1000}}\right] = 1 - \Phi(0.86) = .1949$$

c. $$\beta(.05) = 1 - \Phi\left[\frac{.02 - .05 - 1.645\sqrt{.02(.98)/1000}}{\sqrt{.05(.95)/1000}}\right] = 1 - \Phi(-5.41) \approx 1$$, so the chance the inventory will be *postoned* is P(reject H_0 when p = .05) = 1 – β(.05) = 0. It is highly unlikely that H_0 will be rejected, and the inventory will almost surely be carried out.

42. The hypotheses are H_o: p = .10 vs. H_a: p > .10, so R has the form {c, …, n}. For n = 10, c = 3 (i.e. R = {3, 4, …, 10}) yields $\alpha = 1 - B(2; 10, .1) = .07$ while no larger R has $\alpha \le .10$; however $\beta(.3) = B(2; 10, .3) = .383$. For n = 20, c = 5 yields $\alpha = 1 - B(4; 20, .1) = .043$, but again $\beta(.3) = B(4; 20, .3) = .238$. For n = 25, c = 5 yields $\alpha = 1 - B(4; 25, .1) = .098$ while $\beta(.7) = B(4; 25, .3) = .090 \le .10$, so n = 25 should be used.

46. In each case the p-value = $1 - \Phi(z)$

a. .0778

b. .1841

c. .0250

d. .0066

e. .5438

48.

a. In the df = 8 row of table A.5, $t = 2.0$ is between 1.860 and 2.306, so the p-value is between .025 and .05: $.025 < \text{p-value} < .05$.

b. $2.201 < | -2.4 | < 2.718$, so $.01 < \text{p-value} < .025$.

c. $1.341 < | -1.6 | < 1.753$, so $.05 < P(t < -1.6) < .10$. Thus a two-tailed p-value: $2(.05 < P(t < -1.6) < .10)$, or $.10 < \text{p-value} < .20$

d. With an upper-tailed test and $t = -.4$, the p-value $= P(t > -.4) > .50$.

e. $4.032 < t=5 < 5.893$, so $.001 < \text{p-value} < .005$

f. $3.551 < | -4.8 |$, so $P(t < -4.8) < .0005$. A two-tailed p-value $= 2[P(t < -4.8)] < 2(.0005)$, or p-value $< .001$.

52.

a. For testing H_o: $p = .2$ vs H_a: $p > .2$, an upper-tailed test is appropriate. The computed Z is $z = .97$, so p-value = $1-\Phi(.97)=.166$. Because the p-value is rather large, H_o would not be rejected at any reasonable α (it can't be rejected for any $\alpha < .166$), so no modification appears necessary.

b. With $p = .5$, $1-\beta(.5)=1-\Phi[(-.3+2.33(.0516))/.0645]=1-\Phi(-2.79)=.9974$