Assignment 5 :

Chapter 3: Questions 28, 34, 36, 38, 46, 52, 56, 62

28.

a. E (X) = $\sum_{x=0}^{4} x \cdot p(x)$
= (0)(.08) + (1)(.15) + (2)(.45) + (3)(.27) + (4)(.05) = 2.06

b. V(X) = $\sum_{x=0}^{4} (x-2.06)^2 \cdot p(x)$ = $(0 - 2.06)^2(.08) + \ldots + (4 - 2.06)^2(.05)$
= .339488+.168540+.001620+.238572+.188180 = .9364

c. σ_x = $\sqrt{.9364} = .9677$

d. V(X) = $\left[\sum_{x=0}^{4} x^2 \cdot p(x)\right] - (2.06)^2$ = 5.1800 – 4.2436 = .9364

34. Let h(X) denote the net revenue (sales revenue – order cost) as a function of X. Then $h_3(X)$ and $h_4(X)$ are the net revenue for 3 and 4 copies purchased, respectively. For x = 1 or 2 , $h_3(X) = 2x - 3$, but at x = 3,4,5,6 the revenue plateaus. Following similar reasoning, $h_4(X) = 2x - 4$ for x=1,2,3, but plateaus at 4 for x = 4,5,6.

x	1	2	3	4	5	6
$h_3(x)$	-1	1	3	3	3	3
$h_4(x)$	-2	0	2	4	4	4
p(x)	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{3}{15}$	$\frac{4}{15}$	$\frac{3}{15}$	$\frac{2}{15}$

$E[h_3(X)]$ = $\sum_{x=1}^{6} h_3(x) \cdot p(x)$ = (-1)($\frac{1}{15}$) + … + (3)($\frac{2}{15}$) = 2.4667

Similarly, $E[h_4(X)]$ = $\sum_{x=1}^{6} h_4(x) \cdot p(x)$ = (-2)($\frac{1}{15}$) + … + (4)($\frac{2}{15}$) = 2.6667

Ordering 4 copies gives slightly higher revenue, on the average.

36. E(X) = $\sum_{x=1}^{n} x \cdot \left(\frac{1}{n}\right) = \left(\frac{1}{n}\right)\sum_{x=1}^{n} x = \frac{1}{n}\left[\frac{n(n+1)}{2}\right] = \frac{n+1}{2}$

$E(X^2)$ = $\sum_{x=1}^{n} x^2 \cdot \left(\frac{1}{n}\right) = \left(\frac{1}{n}\right)\sum_{x=1}^{n} x^2 = \frac{1}{n}\left[\frac{n(n+1)(2n+1)}{6}\right] = \frac{(n+1)(2n+1)}{6}$

So V(X) = $\dfrac{(n+1)(2n+1)}{6} - \left(\dfrac{n+1}{2}\right)^2 = \dfrac{n^2-1}{12}$

38. E(X) = $\sum_{x=1}^{4} x \cdot p(x)$ = 2.3, E(X^2) = 6.1, so V(X) = 6.1 – $(2.3)^2$ = .81

Each lot weighs 5 lbs, so weight left = 100 – 5x.
Thus the expected weight left is 100 – 5E(X) = 88.5,
and the variance of the weight left is
V(100 – 5X) = V(-5X) = 25V(x) = 20.25.

46. X ~ Bin(25, .05)

a. P(X ≤ 2) = B(2;25,.05) = .873

b. P(X ≥ 5) = 1 - P(X ≤ 4) = 1 – B(4;25,.05) = .1 - .993 = .007

c. P(1 ≤ X ≤ 4) = P(X ≤ 4) – P(X ≤ 0) = .993 - .277 = .716

d. P(X = 0) = P(X ≤ 0) = .277

e. E(X) = np = (25)(.05) = 1.25
V(X) = np(1 – p) = (25)(.05)(.95) =1.1875
σ_x = 1.0897

52. X ~ Bin (25, .02)

a. P(X=1) = 25(.02)$(.98)^{24}$ = .308

b. P(X≥1) = 1 – P(X=0) = 1 – $(.98)^{25}$ = 1 - .603 = .397

c. P(X≥2) = 1 – P(X≤1) = 1-P(X=0)-P(X=1) = 1-(1-0.397)-0.308= 0.089

d. $\bar{x} = 25(.02) = .5$; $\sigma = \sqrt{npq} = \sqrt{25(.02)(.98)} = \sqrt{.49} = .7$
$\bar{x} + 2\sigma = .5 + 1.4 = 1.9$ So P(0 ≤ X ≤ 1.9) = P(X≤1) = .911

e. $\dfrac{.5(4.5) + 24.5(3)}{25} = 3.03$ hours

56. $h(x) = 1 \cdot X + 2.25(25 - X) = 62.5 - 1.5X$, so $E(h(X)) = 62.5 - 1.5E(x)$
$= 62.5 - 1.5np - 62.5 - (1.5)(25)(.6) = \$40.00$

62.

a. Let X = the number with reservations who show, a binomial r.v. with n = 6 and p = .8. The desired probability is
$P(X = 5 \text{ or } 6) = b(5;6,.8) + b(6;6,.8) = .3932 + .2621 = .6553$

b. Let h(X) = the number of available spaces. Then

When x is:	0	1	2	3	4	5	6
H(x) is:	4	3	2	1	0	0	0

$E[h(X)] = \sum_{x=0}^{6} h(x) \cdot b(x;6,.8) = 4(.000) + 3(.002) + 2(.015) + 1(.082) = 0.117.$

c. Possible X values are 0, 1, 2, 3, and 4. X = 0 if there are 3 reservations and none show or … or 6 reservations and none show, so
$P(X = 0) = b(0;3,.8)(.1) + b(0;4,.8)(.2) + b(0;5,.8)(.3) + b(0;6,.8)(.4)$
$= .0080(.1) + .0016(.2) + .0003(.3) + .0001(.4) = .0013$
$P(X = 1) = b(1;3,.8)(.1) + \ldots + b(1;6,.8)(.4) = .0172$
$P(X = 2) = .0906$, $P(X = 3) = .2273$,
$P(X = 4) = 1 - [\,.0013 + \ldots + .2273\,] = .6636$