Assignment 9:

Chapter 5: 52, 54, 56

52. X ~ N(10), n =4

$\mu_{T_0} = n\mu = (4)(10) = 40$ and $\sigma_{T_0} = \sigma\sqrt{n} = (2)(1) = 2,$

We desire the 95th percentile: 40 + (1.645)(2) = 43.29

54.

a. $\mu_{\overline{X}} = \mu = 2.65$, $\sigma_{\bar{x}} = \frac{\sigma_x}{\sqrt{n}} = \frac{.85}{5} = .17$

P($\overline{X}$ ≤ 3.00)= $P\left(Z \le \frac{3.00 - 2.65}{.17}\right) = P(Z \le 2.06) = .9803$

P(2.65 ≤ $\overline{X}$ ≤ 3.00)= $= P(\overline{X} \le 3.00) - P(\overline{X} \le 2.65) = .4803$

b. P($\overline{X}$ ≤ 3.00)= $P\left(Z \le \frac{3.00 - 2.65}{.85/\sqrt{n}}\right) = .99$ implies that $\frac{.35}{85/\sqrt{n}} = 2.33,$ from which n = 32.02. Thus n = 33 will suffice.

56.

a. With Y = # of tickets, Y has approximately a normal distribution with $\mu = \lambda = 50$, $\sigma = \sqrt{\lambda} = 7.071$,

so P(35 ≤ Y ≤ 70) ≈ $P\left(\frac{34.5 - 50}{7.071} \le Z \le \frac{70.5 - 50}{7.071}\right)$ = P(-2.19 ≤ Z ≤ 2.90) = .9838

b. Here $\mu = 250$, $\sigma^2 = 250, \sigma = 15.811$,

so P(225 ≤ Y ≤ 275) ≈ $P\left(\frac{224.5 - 250}{15.811} \le Z \le \frac{275.5 - 250}{15.811}\right)$ = P(-1.61 ≤ Z ≤ 1.61) = .8926

Chapter 6: 8, 20

8.

a. With q denoting the true proportion of defective components,

$\hat{q} = \frac{(\#defective.in.sample)}{sample.size} = \frac{12}{80} = .150$

b. P(system works) = p^2 , so an estimate of this probability is $\hat{p}^2 = \left(\frac{68}{80}\right)^2 = .723$

20.

a. We wish to take the derivative of $\ln\left[\binom{n}{x} p^x (1-p)^{n-x}\right]$, set it equal to zero and solve for p.

$\frac{d}{dp}\left[\ln\binom{n}{x} + x\ln(p) + (n-x)\ln(1-p)\right] = \frac{x}{p} - \frac{n-x}{1-p}$; setting this equal to zero and solving for p yields $\hat{p} = \frac{x}{n}$. For n = 20 and x = 3, $\hat{p} = \frac{3}{20} = .15$

b. $E(\hat{p}) = E\left(\frac{X}{n}\right) = \frac{1}{n}E(X) = \frac{1}{n}(np) = p$; thus $\hat{p}$ is an unbiased estimator of p.

c. $(1-.15)^5 = .4437$