

# The Game of Fifteen and Other Fun Problems

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## The Rules of the Game:

The numbers 1, 2, 3, 4, 5, 6, 7, 8, 9 are written on the board.

Two players take turns selecting one number from the board. Once a number is chosen, it is crossed off. The winner is the first person to select three numbers that add up to 15. (This game can end in a draw). How do you play the game?

## Analysis of the Game:

As we discovered together, the Game “Fifteen” is completely identical to Tic-Tac-Toe. Not just similar, but *completely identical*. In the process, we learned about magic squares and how to construct a 3 by 3 magic square.

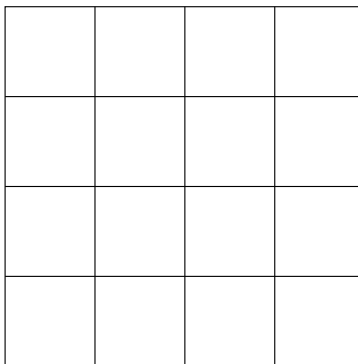
We have employed a powerful mathematical problem-solving strategy known as an *isomorphism*. The Fifteen game is *isomorphic* to Tic-Tac-Toe. That means, solving the Fifteen game is equivalent to solving Tic-Tac-Toe. And since we are familiar with how to play Tic-Tac-Toe, this gives us a strategy to play Fifteen.

Mathematicians often apply this powerful strategy in their research. Sometimes we are able to take a difficult problem, and show that it is equivalent to a much simpler problem that we already know how to solve. Or, we can take a problem from a different field (e.g. molecular biology), convert it into an equivalent math problem, solve the math problem, and convert it back to answer a question about biology. Needless to say, mathematicians have contributed a great deal to the development of physics, economics, chemistry, and every other subject in the sciences.

## Other Problems:

Here are some other fun problems that we will look at to close the session.

1. Determine the number of rectangles in the following diagram.



(Remember that squares count as rectangles too!)

2. There are three girls and three boys who sit down in a row, with one empty chair in the middle.

G G G – B B B

They play a game called “Frogs”. Here are the rules.

The girls may move only to the right, and the boys may move only to the left. They can move in two ways. They can slide into an adjacent empty chair or leap over one person of the opposite gender into an empty chair. No other moves are allowed.

How many moves are needed to get all the boys to the left and all the girls to the right?

B B B – G G G

Now what if you have four girls and four boys. How many moves are required then? How about ten girls and ten boys?

3. Here is a conversation between two good friends, Al and Bob.

**Al:** “I have three kids. The product of their ages is 36”.

**Bob:** “What is the sum of their ages?”

**Al:** “The sum of their ages is the same as my house number”.

**Bob:** “That’s still not enough information to figure out how old they are!”

**Al:** “I’ll give you one more clue. My youngest child absolutely loves chocolate ice cream.”

**Bob:** “Now I know how old your three kids are.”

How old are Al’s kids?

# Fun Warmup Problems

Fill the missing letters!

For example, 18 H on a G C is “18 Holes on a Golf Course”.

90 D in a R A

7 W of the W

29 D in F of a L Y

54 C in a D (with the J)

9 P in the S S

88 K on a P

10 P and 3 T in C

200 D for P G in M

23 P of C in the H B

2 S of R in a P of K R B