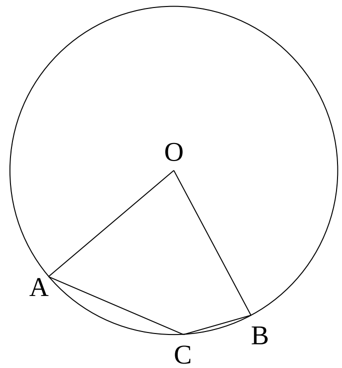
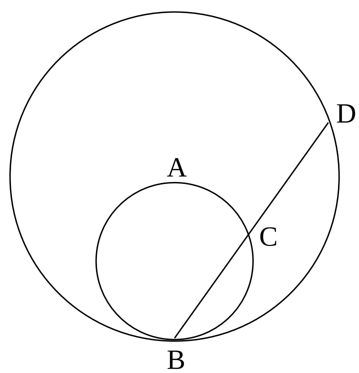
Tour 18 - Problems Involving Circles

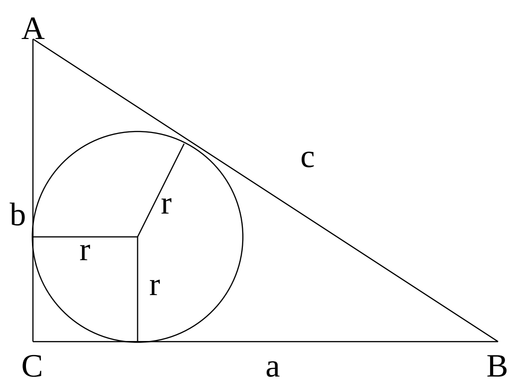
1. In the diagram, O is the centre of the circle, and $\angle AOB = 100°$. Determine $\angle ACB$.



2. In the diagram, AB is a diameter of the small circle and a radius of the large circle that has its centre at A. Prove that $BC = CD$.



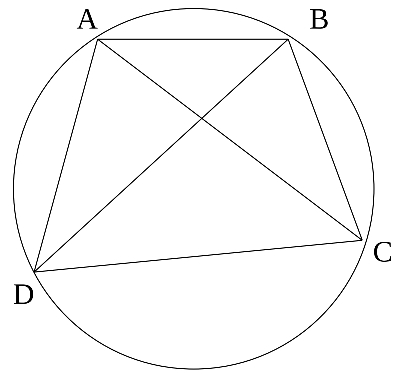
3. $\triangle ABC$ is right-angled, with $\angle C = 90°$. Let r be the radius of the incircle of $\triangle ABC$. Determine r in terms of a, b, and c.



4. Prove the Pythagorean Theorem using the Power of a Point Theorem.

5. Let $ABCD$ be a cyclic quadrilateral. Prove Ptolemy's Theorem:

$$AD \cdot BC + AB \cdot CD = AC \cdot BD$$



(Hint: construct point E on BD such that $\angle AED = \angle ABC$.

6. Prove the Pythagorean Theorem using the Pythagorean Theorem.

7. Across the horizontal bottom of a rectangular wine rack $PQRS$ there is room for more than three bottles (A, B, and C), but not enough for a fourth bottle. All the bottles that are put into this rack are the same size. Naturally, bottles A and C are laid against the sides of the rack and a second layer, consisting of just two bottles (D and E), holds B in place somewhere between A and C. Now we can lay in a third row three bottles (F, G, and H), with F and H resting against the sides of the rack. Then a fourth layer is held with just two bottles (I and J).

   Now if the bottles are not evenly spaced in the bottom row, the second, third, and fourth rows can slope considerably, tilting at different angles for different spacings.

   Prove, however, that whatever the spacing in the bottom row is, the fifth row is always perfectly horizontal!