Solution for Weekly Proof 11

Since the three darts are thrown independently, they each have a $\frac{1}{3}$ chance of being the best throw. As long as the third dart is not the best throw, it will be worse than the second dart (since the second throw is better than the first). Therefore the probability that the third throw is better than the second throw is $\frac{1}{3}$, i.e., the probability that the third throw is the best.

We can also analyze the problem as follows:

Ranking the three darts' results from A (best) to C (worst), there are, a priori, six equiprobable outcomes.

Possibility	**1**	**2**	**3**	**4**	**5**	**6**
First Throw	A	A	B	B	C	C
Second Throw	B	C	A	C	A	B
Third Throw	C	B	C	A	B	A

The information from the first two throws shows us that the second throw will not be the worst, nor the first throw the best.

Thus possibilities 1,2,and 4 are eliminated, leaving cases, 3,5,6, all of which are equally probable. Of these, 3,5 have the third throw worse than the second; in 6, the third throw is better than the second. So the desired answer is $\frac{1}{3}$.

Here is a third solution.

This problem is completely identical to the Monty Hall "Let's Make a Deal" problem. It's as if the best throw is the car, and the other two throws are the goats.

You pick one of the doors (third throw) that you don't know anything about. And Monty tells you nothing about this door. Monty then opens one of the doors, revealing a goat (first throw). That's the exact same thing as saying the second throw is better than the first, i.e., the first throw (or the first door) must be one of the goats.

Now Monty gives you the chance to switch. If you switch, i.e., go to the second throw, your chances of winning are $\frac{2}{3}$. In other words, two out of three times your second throw will be better than your third, given this information.

If you stay, then the chances of winning are $\frac{1}{3}$. In other words, one out of three times your third throw will be better than your second. So the desired answer is $\frac{1}{3}$.