Solution for Weekly Proof 2

Let's look at the slopes of the triangles in the diagram. We'll notice something very fishy here.

The two smaller triangles don't have the same slope as the large triangle. The large triangle has slope $\frac{5}{13}$, but the slopes of triangles 1 and 2 are $\frac{3}{8}$ and $\frac{2}{5}$, respectively.

Note that these three fractions are very close, but are not the same. But the slopes are close enough to fool the human eye into thinking that these slanted lines are all running on top of each other. By the way, do you think it's a coincidence that the numbers that appear in our slopes (namely, 2, 3, 5, 8, and 13) are Fibonacci numbers? The answer is no, it's not a coincidence!

If you look carefully at the first diagram, you will see that there is a gap between the edges of the two little triangles and the whole-area one. That gap has an area of 0.5 square units. Note, that comes from the area of the whole triangle ($5 * 13/2 = 32.5$) minus the areas of the four individual pieces ($12 + 5 + 7 + 8 = 32$).

Now look at the second diagram. When the triangle pieces are switched, they cover not only the 0.5 square unit gap shown in the first diagram, but also extend outside the whole area triangle and cover an area of 0.5 square units there. (Do you see why that's true?)

Since the puzzle pieces cannot cover any more area that 32 square units, the "hole" appears. The hole came from the 0.5 square unit "gap" and the 0.5 square unit "overlap" covered by the triangular puzzle pieces. So that is where the missing square comes from!