Proof for Week 4

Due on Thursday, October 4^{th}

In today's tour, we showed that if $M_k = 2^k - 1$ is a prime number (i.e. M_k is a Mersenne prime), then the number $2^{k-1}M_k$ is perfect. This result was proved by Euclid in 300 B.C.

It turns out that the converse is true: if the integer n is an even perfect number, then it *must* be of the form $2^{k-1}M_k$ where M_k is a Mersenne prime. This was not proved until the great mathematician Euler did, sometime in the eighteenth century.

Euler is considered to be one of the three greatest mathematicians in history (the other two being Archimedes and Gauss), due to his vast contributions to numerous fields of mathematics. The non-mathematics world has also recognized his achivements. For example, the city of Edmonton decided to name their hockey team after him: The Edmonton "Eulers".

For this Weekly Proof, you will be asked to prove Euler's result. You are to prove that *if* n is an even perfect number, *then* n must be of the form $2^{k-1}(2^k - 1)$, where $2^k - 1$ is prime.

Note: this is extremely extremely hard. A complete proof will get you three points, rather than the usual two. Lots of part marks will be awarded for good ideas. So you can get two points just by making a little bit of progress with the problem. Thus, if you make ANY sort of attempt on this problem, hand in what you have, and you will definitely receive some marks.