Topics in Graph Theory – Problem set 3

Due Tuesday, Feb. 4, beginning of class

1. True or false: In every k-critical graph, every vertex has degree at least $k-1$. If true, give a proof, if false, give a counterexample.

2. A split graph is a graph whose vertices can be partitioned into a clique and an independent set.

   (a) Draw an example of a split graph

   (b) Show that the complement of a split graph is again a split graph. (The *complement* of a graph $G = (V, E)$ is the graph with vertex set V where two vertices u, v are adjacent in the complement precisely when they are not adjacent in G.)

   (c) Show that split graphs are perfect.

3. Let G be the complement of a connected, bipartite graph. What is $\alpha(G)$? (b) Show that G is perfect, using only the definition of perfection. **Do not use the theorems shown in class Jan. 23 and 28.** *Hint: Use one of the theorems about matchings.* BONUS: where does your proof go wrong if the graph is not connected?

4. (MATH 5330) A *division graph* is defined as follows: the vertex set is a set of positive integers, and vertex i is adjacent to j if and only if i divides j or j divides i.

   (a) What can you say about the integers that form a clique?

   (b) Show that division graphs are perfect.

5. (MATH 4330/CSCI 4115) We know that bipartite graphs are perfect. Give an *algorithm* that finds, for any demand vector s, a perfect graph colouring of (G, s) if G is a bipartite graph. Prove carefully that your algorithm uses the minimum number of colours. (This was discussed in class.)