Topics in Graph Theory – Problem set 8

Due Tuesday, March 18, beginning of class

1. (a) Let G be a d-regular graph with n vertices. Give a lower bound on the diameter. Use the approach seen in class on March 11, so start with a vertex, and consider its neighbours, its neighbours' neighbours, etc. *Note that this is a deterministic question, so no probability involved!* (b) Consider $G(n,p)$ where $p = n^{\alpha}$ for some $\alpha \in (0,1)$. Give an estimate for the diameter, based on the approach used in (a) and in class on March 11.

2. Consider digraphs where there is at most one arc between any two vertices, so digons do not occur. Let G be such a digraph, which is minimally strongly connected, and has n vertices. Give and prove an upper bound for the number of edges in G, and give a graph of more than 4 vertices which achieves this upper bound.

3. Consider the directed version of $G(n,p)$, where each possible arc (u,v) is added with probability p. (a) Give the expected number of digons in this graph. (b) Show that, if p is a constant in the interval $(0,1)$, is a.a.s strongly connected. *Hint: prove the stronger statement that for any pair of vertices u, v, there is a directed path of length at most 2 from u to v.*