List Chromatic Number of Complete Bipartite Graphs

Emma Graham

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Proposition: (Erdös – Rubin – Taylor [1979])

If $m = (2k-1)C(k)$, then $K_{m,m}$ is not k-choosable

Proof:

Let X,Y be the bipartition of $G = K_{m,m}$. Assign distinct k-subsets of [2k-1] as the lists for the vertices of X and do the same for Y. Consider a choice function, f. If f uses fewer than k distinct choices in X, then there is a k-set $S \subseteq [2k-1]$ not used.

Which means that no colour was chosen for vertex of X having S as its list. If f uses at least k colours of X, then there is a k-set $S \subseteq [2k-1]$ of colours used in X, and no colour can be properly chosen for vertex of Y with list S.