MATH 4370/5370

Study guide for final exam

The final exam will consist of an in-class part and a take-home part. For the take-home part, you may consult any material used in the course, and the books referred to one the Web page. Do not consult any other sources, and do not talk to anyone about the questions.

The in-class part will be closed book, and will have basic questions. You will have 90 minutes, but I will aim to keep the exam so that it can be completed in 60 minutes or less.

Both the in-class and the take-home will cover all the material in the course, including that which was covered on the midterm. However, the emphasis will be on the later material. Below is a list of the topics you should be familiar with, starting with the later material. Consult the list of class topics on the Web page to see where this material can be found.

- Ramsey numbers. Definition and recursive upper bound. Two different upper bounds that can be derived with the probabilistic method.
- Generating functions. Manipulations with generating functions: multiplying by x, formal differentiation, reciprocals, inverse. Convolutions and multiplication of generating functions. Significance of a power $f^k(x)$ of a generating function. Significance of $f(x)/(1-x)$ where $f(x)$ is a generating function.
- Applications of generating functions. Deriving a closed formula from a recurrence relation, for example for Fibonacci numbers. Using generating functions to prove combinatorial identities. Partitions. Binomial coefficients. Integer solutions to a linear equation. Sequences with generating functions e^x and $\log(1-x)$. Application to derangements.
- Steiner triple systems. Definition. Small examples: $STS(7)$, $STS(9)$. Proof that STS can only exist for $n \equiv 3 \mod 6$ or $n \equiv 1 \mod 6$. Three different constructions: for prime order, for multiples of 3, and for $n \equiv 1 \mod 6$. For prime order (method of differences), know definition of primitive element in $\mathbb{Z}_p$.
- Designs. Definition and two different notations: $S_\lambda(n,k,t)$ and $t-(n,k,\lambda)$ design. Incidence matrix. How to derive formulas for the number of blocks, and the number of times an element

occurs in a block. Derived and residual design. Symmetric designs and their special parameters. Projective planes.

- Probabilistic method. General idea: if the probability of bad events is smaller than 1, then there must exist one instance of the process without bad events. Applications based on the inequality $P(\bigcup_i A_i) \leq \sum_i P(A_i)$ — the A_i are the bad events. Application to Ramsey numbers, monochromatic arithmetic progressions.
- Random variables. Indicator variables. Linearity of expectation. Using the fact that, if $E(X) = a$, then there must exist one instance where $x \geq a$, and an instance where $X \leq a$. Applications to sum-free sets, Hamiltonian paths in tournaments. With a twist: application to independence number of a graph.
- Latin squares. Mutually orthogonal latin squares. How to construct MOLS for primes. How to construct MOLS for prime powers (irreducible polynomial given). How to make larger MOLS from smaller ones.
- Systems of distinct representatives. Hall's theorem and its proof. Special cases where all sets are the same size etc.. Number of SDRs. Stochastic matrices and their relation with SDRs.
- Asymptotic notation: Big Oh etc. Stirling's formula.
- Inclusion/exclusion formula, and its proof using the binomial theorem. Derangements. Euler ϕ function.
- Pigeonhole principle. Monotone subsequences.
- Partially ordered sets and relevant definitions (chain, antichain, comparable..) Dilworth theorem and proof. Minsky's theorem. Application to prove Sperner's theorem.