Combinatorics – Problem set 7

Due Tuesday, March 31, beginning of class

For details on designs, consult the chapter from the book by van Lint and Wilson uploaded on the course Web page. See also the links to various set of notes.

1. (a) Construct an STS(15), using the general construction for $n \equiv 3 \mod 6$. You do not have to list all blocks explicitly, just give a precise description. (b) Give the incidence matrix N (again, it suffices to give part of the matrix explicitly, and indicate the rest) (c) Give a proof that every pair of points indeed occurs in exactly one triple. (d) Count the number of blocks, and verify that this satisfies the general formula for the number of blocks in an STS.

2. (a) Construct an STS(19), using the general construction for $n \equiv 1 \mod 6$. This is the method where we consider the set of points as 9 sets of three points which all intersect in a common point u. You do not have to list all blocks explicitly, just give a description. (b) Give the incidence matrix N (again, it suffices to give part of the matrix explicitly, and indicate the rest) (c) Give a proof that every pair of points indeed occurs in exactly one triple. (d) Count the number of blocks, and verify that this satisfies the general formula for the number of blocks in an STS.

3. Construct an STS(13), using the method of differences. This method can be described as follows. . (a) First, find a primitive element α in $\mathbb{Z}_{13} \setminus \{0\}$. (b) Give explicitly all the sets $B_i = \{\alpha^i, \alpha^{i+2t}, \alpha^{i+4t}\}$ where $0 \leq i < t$. (c) Give all the differences covered by each of the B_i, and show that they cover all the differences. (d) Give the STS. You do not have to list all blocks explicitly, just give a precise description. (e) Give the incidence matrix N (again, it suffices to give part of the matrix explicitly, and indicate the rest) (f) Give a proof that every pair of points indeed occurs in exactly one triple. (e) Count the number of blocks, and verify that this satisfies the general formula for the number of blocks in an STS.

4. We saw the construction of a $S(3, 4, 10)$ design constructed as follows: the points are the edges of a complete graph on 5 vertices, and the blocks consisted of certain sets of edges (3 types). (a) Give the incidence matrix of this design. (Exactly). Give the derived design and the residual design obtained from fixing one particular point.