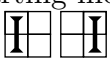
April 2009 Questions

Meghan and Geoff play the following game. On an $m \times n$ board, they each place dominoes which cover two adjacent squares. Meghan's dominoes cover two adjacent vertical squares while Geoff's cover two adjacent horizontal squares. Dominoes cannot be placed on top of another.

Meghan's possible starting moves on a 2×2 board:



Geoff's possible starting moves on a 2×2 board

Figure 1: A 2×2 board with Meghan's possible starting moves and Geoff's possible starting moves.

Play alternates between the two players until there are no moves left to play. The last player to place a domino wins.

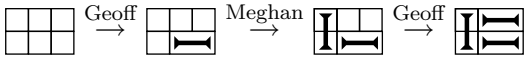


Figure 2: A game on a 2×3 board with Geoff moving first and Geoff winning.

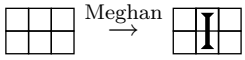


Figure 3: A game on a 2×3 board with Meghan moving first and Meghan winning, since after Meghan's one move, there are no adjacent horizontal squares for Geoff to move in.

Try to find some results about this game. For example, you could find a set of positions where Meghan always wins moving first, or you could analyse some $2 \times n$ boards for different n, or you could give a strategy for certain board sizes (these are just examples, feel free to show something completely different. Anything you want to show is acceptable).

Hint: a good way to discover results about this game is to play it with a friend. Try to see what strategies work or what always seems to happen on

a certain sized board. Formalize these results into proofs.

Solutions will be marked on how well they are presented and explained, not on how many different things you can prove. It is better to submit one well-written and well-explained result than ten sloppy, difficult-to-follow results.

Remember:

1. Define any variable you use.
2. Explain what you are going to do before you do it.
3. Justify your steps, i.e. if something is true for a reason, say what that reason is.
4. Don't submit your first copy. Read it over to check for mistakes and clarity.

Submit all solutions by 23.59 April 30, 2009.