December 2008 Questions

1) Consider the first $2n$ positive integers $1, 2, 3, \ldots, 2n-1, 2n$. Pair off the integers as follows:

$$(1, 2n), (2, 2n-1), (3, 2n-2), \ldots, (n, n+1),$$

and take the product of each of the pairs. Show that each product pair is unique, i.e. not equal to any other product pair.

2) Geoff and Meghan play the following game with a heap of n tokens: A move is to choose a heap of size at least 2 (for the first move, this is only one heap to chose) and split it into two non-empty heaps. The game ends when all the heaps have size 1 and the player who split the last heap is the winner (equivalently, if on a player's move, all the heaps are of size 1, then he/she is the loser and the other player is the winner). If Geoff moves first, determine, for all positive integers n, whether Geoff or Meghan wins the game.

Submit all solutions by 23.59 December 31, 2008.