

February 2009 Questions

- 1) Consider the following polynomial

$$p(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$$

where a_i is an integer for each $i \in \{0, 1, \dots, n\}$. If $p(0)$ and $p(1)$ are both odd, show that $p(x)$ has no integer roots, i.e. there does not exist an integer t such that $p(t) = 0$.

- 2) Suppose five people each know exactly one piece of information and that each of the five pieces of information are different. Every time person A phones person B, A tells B everything that A knows, while B tells A nothing. Find the minimum number of phone calls between pairs of people required for everyone to know everything. You must show that your answer is a minimum.

Submit all solutions by 23.59 February 28, 2009.