Nova Scotia Math League

2012–2013

Game Two

Contest Paper

Team Questions

1. Kelly was born on January 1 sometime in the 20th century, and she will be y years old in the year y^2. How old is Kelly today?

2. Sunny runs at a steady rate, and Moonbeam runs 25% faster. If Moonbeam gives Sunny a head start of 10 metres, how many metres must Moonbeam run before overtaking Sunny?

3. Alan finds himself stranded and needs to call a taxi from a payphone. Unfortunately, he cannot remember the final digit of the cab company's phone number. He is sure of the first six digits, so he decides to guess randomly at the last digit.

   Given that Alan has enough change to make two phone calls, what is the probability that Alan will eventually be successful in calling a cab?

4. Find the next term in the following sequence:

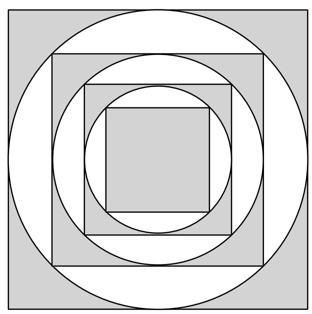
$$3+2\sqrt{2},\quad 7+5\sqrt{2},\quad 17+12\sqrt{2},\quad 41+29\sqrt{2},\quad 99+70\sqrt{2},\ldots$$

5. The area of a rectangular plot of land remains unchanged when the plot is made 15 metres longer and 4 metres narrower, or when it is made 15 metres shorter and 8 metres wider. What is the area of the plot, in square metres?

6. The student lockers at Colossus High are numbered consecutively beginning with locker number 1. The plastic digits used to number the lockers cost five cents each. (So, for instance, it costs 5 cents to label locker #8, and 15 cents to label locker #312.)

   If it costs \$347.25 to label all the lockers at the school, how many lockers are there?

7. In the figure below, the circles and squares have been inscribed in each other in an alternating manner. If the largest (outer) square has an area of 9 square units, what is the area of the smallest (inner) square?



8. What is the largest value of c such that the line $y = 2x + c$ intersects the circle $x^2 + y^2 = 25$?

9. The sum of the base and perpendicular of a right angled triangle is 23, and the sum of the base and hypotenuse is 25. Find the perimeter of the triangle.

10. Find the largest positive integer m such that $m^3 + 3$ is divisible by $m + 3$.

Pairs Relay

P-A. Suppose x, y, z are real numbers such that

$$(x-20)^2+(y+12)^2+(z-24)^2=0.$$

Let $\mathtt{A}=x+y+z$. **Pass on A**

P-B. You will receive A.

A gambler plays a game with even odds. (That is, if he wagers x dollars, then he either wins x dollars or loses x dollars.) He plays the game multiple times in succession, always wagering **half** of his current money.

The gambler begins with A dollars, and after four plays he has won twice and lost twice. Let B be the number of dollars that the gambler **has lost** at this point.

Pass on B

P-C. You will receive B.

Define the operator $\oplus$ by the formula

$$u \oplus v=\frac{u+v}{1+uv}.$$

Find C such that

$$\mathtt{B} \oplus \frac{1}{\mathtt{C}}=9.$$

Pass on C

P-D. You will receive C.

An ant finds itself at the very center of a 10×10 unit grid. The ant begins to walk, travelling only along grid lines and **never retracing its steps**, until it has travelled a total distance of $\sqrt{\mathtt{C}}$ units. Let D be the number of possible points where the ant could end its journey.

Done!

Individual Relay

I-A. Let S be the sum of all possible positive integers that can be written using the digits 1, 2, and 3 exactly once each. (For example, 312 is such a number, while 331 is not.)

Let $\mathtt{A}$ be the **sum of the digits** of S. Pass on $\mathtt{A}$

I-B. You will receive $\mathtt{A}$.

Let $\mathtt{B}$ be the number of fractions in the list

$$\frac{1}{\mathtt{A}}, \frac{2}{\mathtt{A}}, \frac{3}{\mathtt{A}}, \ldots, \frac{50}{\mathtt{A}}$$

that are **not** reduced to lowest terms.

Pass on $\mathtt{B}$

I-C. You will receive $\mathtt{B}$.

A large cube of side length $\mathtt{B}$ is constructed from $\mathtt{B}^3$ unit cubes. All six faces of this large cube are painted red. Let $\mathtt{C}$ be the number of unit cubes that have exactly **two** sides painted red.

Pass on $\mathtt{C}$

I-D. You will receive $\mathtt{C}$.

Suppose $f(1) = 1$ and $f(n) = 2n - f(n-1)$ for all integers $n \geq 2$.

Let $\mathtt{D} = f(\mathtt{C})$. (Hint: Look for a pattern.) Done!