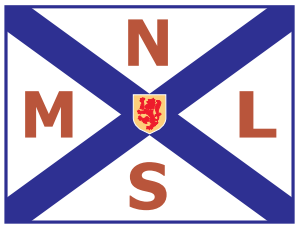


Nova Scotia
Math League

2015–2016

Game Two

Problems

Team Questions

1. In Sparkville, one out of three women is an electrician and two out of three electricians are women. If there are 330 women in Sparkville, how many electricians live in the town?

2. Jane walks 25% faster than her brother Jim. If Jim leaves for school 3 minutes before Jane, how long will it take Jane to catch up?

3. A bank machine only dispenses \$20 and \$50 notes. If John's daily withdrawal limit is \$1000, how many different (nonzero) amounts of money can he withdraw on any given day?

4. A Grade 1 class sits in a circle. The teacher numbers the students $1, 2, 3, \ldots$ starting with Jane and going clockwise. After she is done, Ryan insists that he is number 1 and goes about renumbering everyone (again clockwise) starting with himself. In doing so, Jane's number changes to 10, and her sister Sara's number changes from 14 to 3.

   How many students are in the class?

5. The eight lines

$$\begin{array}{llll} y = x + 1 & y = 2x + 1 & y = 3x + 1 & y = 4x + 1 \\ y = x - 1 & y = 2x - 1 & y = 3x - 1 & y = 4x - 1 \end{array}$$

   are drawn in the (x, y)-plane. Find the number of distinct points of intersection.

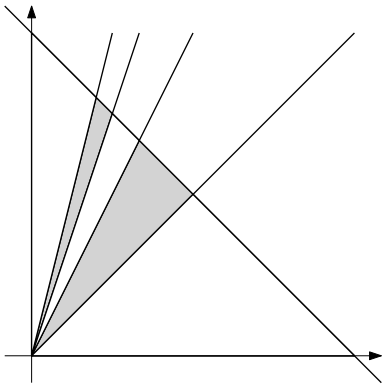
6. Three 6-sided dice — one red, one green and one blue — are thrown. Leting R, G and B denote the values showing, find the probability that $R \leq G \leq B$.

7. The polynomial $f(x) = x^3 + ax^2 + bx + c$ satisfies

$$f(x + 2) = x^3 + 7x + 5 \quad \text{for all } x.$$

   Determine $a + b + c$.

8. The lines $y = x$, $y = 2x$, $y = 3x$ and $y = 4x$ intersect the line $x + y = 1$ to create the figure below.

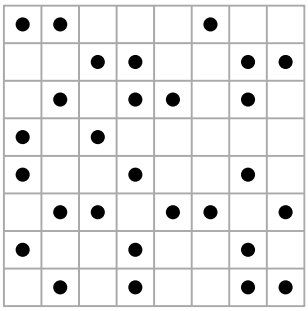


Find the area of the shaded region.

9. Alan, Betty, Clara, Doug, and Elizabeth go on an Easter egg hunt. Altogether the kids find 125 eggs. The three girls have a total of 60, with Elizabeth having more than Betty and Clara combined, and Clara having 5 more than Betty. Alan has more than Doug, but Doug has more than each of the girls.

   How many eggs does Clara have?

10. Kate has placed pennies on an 8×8 grid as shown below.



Kate now wishes to add or remove pennies to make every horizontal row the same. Find the least number of moves (additions and removals) she must perform to accomplish her goal.

Pairs Relay

P-A. Chocolate chip cookies can be purchased in packs of 3 for \$2.50 or packs of 5 for \$3.50. Oatmeal cookies come only in packs of 6 for \$4.

Let A be the most cookies you can purchase for \$35. Pass on A

P-B. You will receive A.

For certain x and y we have $\mathsf{A} : 100 = (2x + y) : (4x + y)$.

Let $\mathsf{B} = \dfrac{x}{y}$. [*Hint:* This should be an integer.] Pass on B

P-C. You will receive B.

Compute:

$$\mathsf{C} = \frac{(\mathsf{B}^2 + 3\mathsf{B} + 2)(\mathsf{B}^2 - 4\mathsf{B} + 3)}{(\mathsf{B}^2 - 3\mathsf{B} + 2)(\mathsf{B}^2 - 2\mathsf{B} - 3)}.$$

Pass on C

P-D. You will receive C.

When the line $y = \mathsf{C}x + \mathsf{D}$ is shifted to the left by D units and shifted down by C units, the resulting line has equation $y = \mathsf{C}x + 2\mathsf{D}$.

Find D. Done!

Individual Relay

I-A. Let A be the number of ways the letters of the word YELL can be arranged so that the two L's are not adjacent.

Pass on A

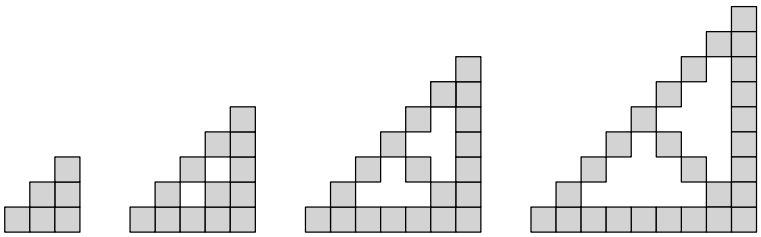
I-B. You will receive A.

The average of a list of A numbers is $A - 1$. Let B be the average after the numbers A and 2A are appended to the list.

Pass on B

I-C. You will receive B.

Let C be the number of squares in the B-th figure of this sequence:



Pass on C

I-D. You will receive C.

Solution X is 15% alcohol by volume and Solution Y is C% alcohol by volume. Let D be the number of litres of Solution X that must be mixed with 5L of Solution Y to result in a 25% alcohol solution.

Done!

Team Questions Answer Key

1. 165
2. 12 minutes
3. 98
4. 20
5. 14
6. $\frac{7}{27}$
7. -4
8. $\frac{13}{120}$
9. 17
10. 24

Pairs Relay Answer Key

A. 52

B. 6

C. 2

D. 2

Individual Relay Answer Key

A. 6

B. 6

C. 41

D. 8