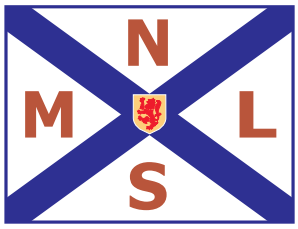


Nova Scotia
Math League

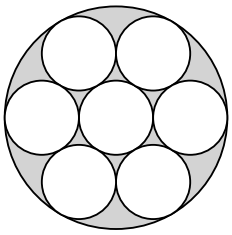
2015–2016

Game Three

PROBLEMS

Team Questions

1. Find the ratio of the shaded to unshaded area in the diagram below:



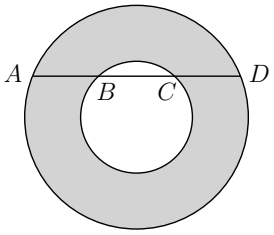
2. John is driving on the highway at 100 km/h. He glances down and is quite pleased to notice that his odometer reads 56965 km, which is palindromic (*i.e.* it reads the same forwards and backwards).

   If John continues driving at the same speed, how many minutes will pass before his odometer again shows a palindrome?

3. Recall that $n! = n(n-1)(n-2)\cdots 3\cdot 2\cdot 1$. For example, $3! = 3\cdot 2\cdot 1 = 6$.

   Find the smallest n such that $n!$ is divisible by 1000.

4. In the figure below, the two circles are concentric and $|AB| = |BC| = |CD| = 2$. Find the area of the shaded region.



5. Find the equation of the line that is obtained by reflecting the line $y = 1$ in the line $y = 2x$.

6. A broken watch gains 8 minutes per hour. If it is set to the correct time at noon, what is the real time when the watch reads 4:15pm?

7. Alan, Bob, and Carl go to Las Vegas to gamble, each bringing a different amount of money. If Alan or Bob were to double their money then the group's total would increase by 25% and 40%, respectively.

   What would be the percent increase if Carl were to triple his money?

8. A square and a rectangle have perimeter 8, but the rectangle has only $\frac{7}{8}$ the area of the square. Find the length of the diagonal of the rectangle.

9. Find the sum of the digits of $(100000001)^5$.

10. An ant paces along the x-axis at a constant rate of one unit per second. He begins at $x = 0$ and his path takes him one unit forward, then two back, then three forward, etc.

    How many times does the ant step on the point $x = 10$ in the first five minutes of his walk?

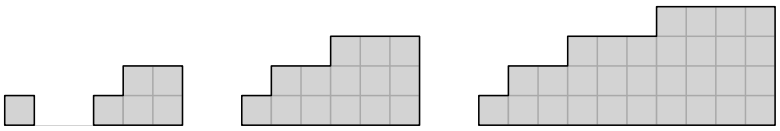
Pairs Relay

P-A. One litre of wine is poured into five litres of water. Five litres of this solution is then mixed with one litre of wine.

Let A be the ratio of wine to water in the final solution. Pass on A

P-B. You will receive A. Let $n = 25\mathsf{A}$, which should be an integer.

Let B be the perimeter of the n-th shape in the following series:



(Each of the small squares in the figure is 1×1.) Pass on B

P-C. You will receive B.

Let C be the smallest positive integer such that $1 + 2 + 3 + \cdots + \mathsf{C}$ is larger than B.

Pass on C

P-D. You will receive C.

Alan and Bill are in a race, each running at constant speed. At 1:00pm, Alan is C metres ahead of Bill, and at 1:15pm he has tripled that lead. Alan finishes the race at 2:00pm.

Let D be the number of metres by which Alan beats Bill. Done!

Individual Relay

I-A. The sum of the lengths of all edges of a cube is 24.

Let A be the surface area of this cube. Pass on A

I-B. You will receive A.

Evaluate:

$$B = (2+4+6+8+\cdots+4A)-(1+3+5+7+9+\cdots+(4A-1))$$

. Pass on B

I-C. You will receive B.

Three consecutive positive integers sum to B.

Let C be the largest of these three integers. Pass on C

I-D. You will receive C.

The lines $y = 1 + Cx$ and $y = 1 - Cx$ intersect the line $y = C$ at two points.

Let D be the distance between these points. Done!

Team Questions Answer Key

1. $\frac{2}{7}$
2. 66 minutes
3. 15
4. 8π
5. $3y + 4x = 5$ or $y = -\frac{4}{3}x + \frac{5}{3}$
6. 3:45pm
7. 70%
8. 3
9. 14
10. 5

Pairs Relay Answer Key

A. 11 : 25

B. 154

C. 18

D. 162

Individual Relay Answer Key

A. 24

B. 48

C. 17

D. $\frac{32}{17}$