

Nova Scotia

Math League

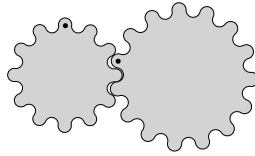
2017–2018

Game Three

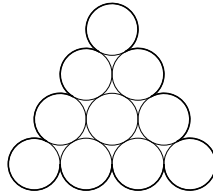
PROBLEMS

Team Questions

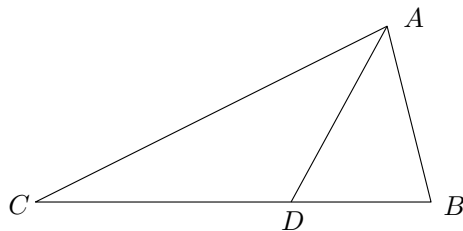
1. Find the percentage increase in the quantity x^2/y^2 when x is increased by 8% and y is decreased by 10%.
2. The smaller gear below begins to turn at $\frac{1}{4}$ revolution per minute. How many minutes elapse before both gears return to their starting positions?



3. Each of the circles in the figure below has circumference 1. Find the perimeter of the figure (highlighted in the diagram).

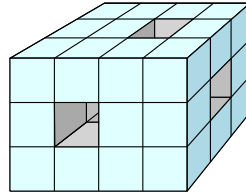


4. It takes 5 minutes to fill my bathtub and 7 minutes for it to drain empty. How many minutes will it take to fill the bathtub if I leave the drain open while filling it?
5. Triangle $\triangle ABC$ is isosceles, with $|AC| = |BC|$. The bisector of $\angle CAB$ meets BC at D , and $\angle ADB = 3\angle ACB$.



Find the degree measure of $\angle ACB$.

6. A $3 \times 3 \times 4$ block is created by gluing together several unit cubes. Three 1×1 square tunnels are then bored completely through the cube as shown below. (The tunnels are perpendicular to the faces.)



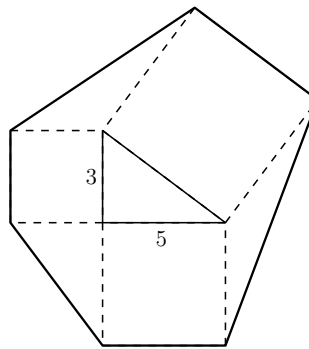
Find the surface area of this solid.

7. Eight friends want to split into 4 pairs to play a game. In how many ways can this be done?
8. For how many integers n between 1 and 100 (inclusive) is

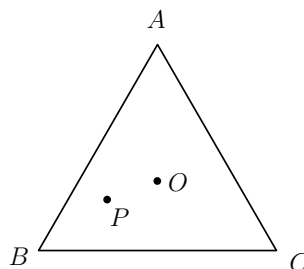
$$2018^n - 2017^n$$

divisible by 5?

9. Squares are extended from the sides of a right triangle with legs of lengths 3 and 5. The vertices of the square are then adjoined to form an irregular hexagon, as shown. Find the area of the hexagon.

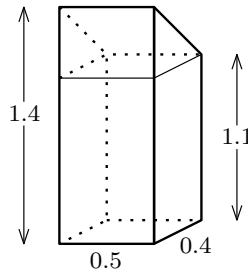


10. Let O be the centre of equilateral triangle $\triangle ABC$ (i.e. the unique point equidistant from each vertex). Another point P is selected at random in the interior of $\triangle ABC$. Find the probability that P is closer to O than it is to any of A, B or C .



Pairs Relay

P-A. A simplified schematic of a mailbox is shown below, with all measurements in metres.



Let A be the volume of the mailbox, in cubic metres.

Pass on A

P-B. You will receive A . Let $n = 40A$. (This should be an integer.)

Let B be the units digit of 123^n .

Pass on B

P-C. You will receive B .

Suppose $x + y = B$, $x + z = 2B$ and $y + z = 3B$.

Let $C = x + y + z$.

Pass on C

P-D. You will receive C .

Jessica plans to give each of her friends a bag of 30 candy hearts for Valentine's Day. She labels some bags (one for each friend) and begins filling them one at a time. Unfortunately, she finds that she only has C hearts remaining for the last bag. So she instead decides to give each friend only 29 hearts and she keeps the 10 leftovers for herself.

Let D be the number of hearts Jessica began with.

Done!

Individual Relay

I-A. Colin, John, and Marc went camping. Over the course of the trip, Marc paid \$93 for food, Colin paid \$58 for gas, and John paid \$53 for the campsite. Both John and Colin gave Marc some money so as to split the costs of the trip equally.

Let A be the amount (in dollars) John gave Marc.

Pass on A

I-B. You will receive A .

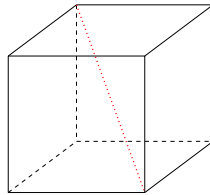
A 250 metre long train travels at a constant speed of 90 km/h. The train enters a tunnel and fully emerges A seconds later.

Let B be the length of the tunnel, in metres.

Pass on B

I-C. You will receive B .

A *main diagonal* of a cube connects two opposing vertices, as shown:



Suppose the volume of a cube is $B \text{ cm}^3$. Let C be the length (in cm) of its main diagonal, **rounded to the nearest integer**.

Pass on C

I-D. You will receive C .

Suppose $x : y = 2 : 1$ and $y : z = C : 2$.

Let $D = \frac{y}{x + z}$.

Done!

Team Questions Answer Key

1. 44
2. 20
3. $\frac{11}{2}$
4. $\frac{35}{2}$
5. 20°
6. 88
7. 105
8. 50
9. 98
10. $\frac{2}{3}$

Pairs Relay Answer Key

- A. $\frac{1}{4}$ (or 0.25)
- B. 9
- C. 27
- D. 387

Individual Relay Answer Key

- A. 15
- B. 125
- C. 9
- D. $\frac{9}{20}$