

Nova Scotia

Math League

2018–2019

Game One

PROBLEMS

Team Questions

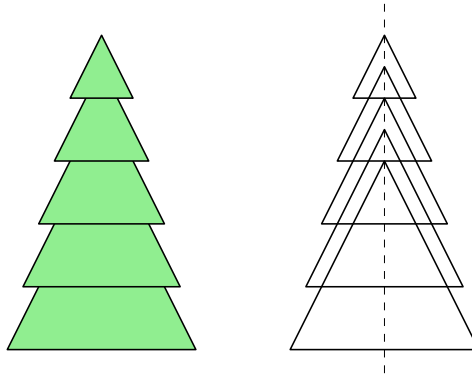
1. Let

$$S = 1 + 2 + 3 + \cdots + 300$$

$$T = 4 + 8 + 12 + \cdots + 600.$$

Compute $T - S$.

2. Paper Christmas trees can be made by cutting out several similar isosceles triangles, each with base equal to its height, and then laying them down in decreasing order of size so that their altitudes align along a common axis and the top of each triangle lies above that of the last.



Find the area of the tree formed from five triangles of heights 2, 3, 4, 5, and 6 cm with their tops uniformly spaced 1 cm apart.

3. Six friends have a race. Bob places second. Jim finishes ahead of Andy and Nancy but behind Ray. Nancy finishes behind Todd. Assuming there were no ties, in how many different orders could they have finished?
4. A bus begins its journey at the mall. At its first stop, a third of the passengers get off while 8 new passengers get on. At the next stop, a third of the passengers again get off while 10 get on. At this point there are 25% fewer people on the bus than there were at the beginning of the journey.

How many people were on the bus as it left the mall?

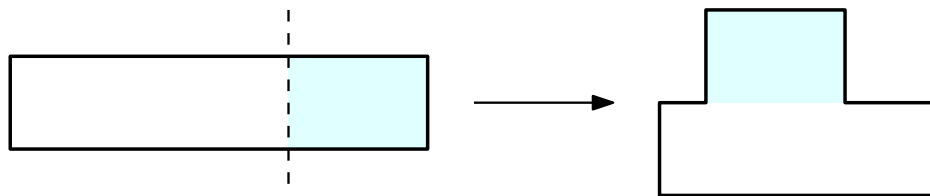
(*Hint*: Don't forget the driver!)

5. Alan and Beth run around an oval track, starting at noon from the same position and each running at constant speed in the same direction. Alan takes 2 minutes to complete a lap, while Beth takes 15 seconds less.

If they run for 1 hour, when does Beth overtake Alan for the last time?

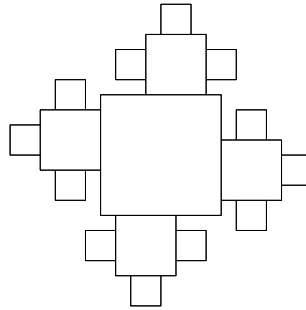
(Give your answer in HH:MM:SS format.)

6. Three six-sided dice are rolled. Find the probability that the sum of the two smallest numbers showing is less than the third.
7. The quadratic $3x^2 + 12x + 7 = 0$ has roots p and q . Find the unique pair (a, b) such that $x^2 + ax + b$ has roots $2p - q$ and $2q - p$.
8. A rectangle with perimeter 20 and area 16 is cut parallel to its shortest side to create two unequal rectangles. The smaller of these rectangles is then translated so that it abuts the top of the larger, as shown below. If the perimeter of this new figure is 19, what is the area of the smaller rectangle?



Pairs Relay

P-A. The figure below was created from several squares having perimeters 1, 2, and 5.



Let A be the overall perimeter of the figure.

Pass on A

P-B. You will receive A .

The parabolas $y = x^2 + 3x + A$ and $y = 2x^2 + 4x + 9$ cross at two points. Let B be the y -intercept of the line joining these points.

Pass on B

P-C. You will receive B .

The elements of the set $\{B + 20, 3B - 50, 75 - 2B\}$ are taken in pairs and the average of each pair is calculated.

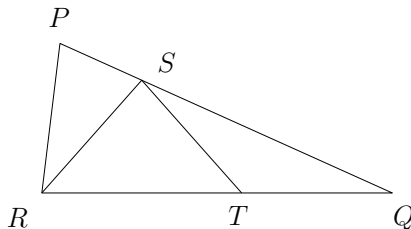
The results are $C - 2$, $C + 2$, and $C + 12$, in no particular order.

Determine C .

Pass on C

P-D. You will receive C .

In the figure below, $\angle PQR = C^\circ$ and $|PR| = |RS| = |ST| = |TQ|$.



Let D be the degree measure of $\angle PRS$.

Done!

Individual Relay

I-A. The *CVV code* on a credit card is a sequence of three digits (0 permitted) that is used for security purposes. The CVV on my card happens to have **distinct** digits, two of which are 4 and 7.

Let A be the number of possible CVV codes on my card.

Pass on A

I-B. You will receive A .

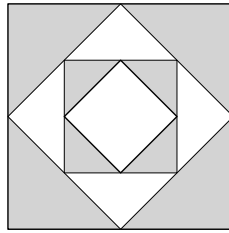
John is struggling with math. His average score on the first three tests was $A\%$. Then he did so horribly on the next test that his average over the first four sunk to 40% .

Let B be John's percentage score on the fourth test.

Pass on B

I-C. You will receive B .

The figure below was created by joining the midpoints of each side of a square to create a smaller square, and then repeating this process twice more. The largest square has area B .



Let C be the area of the shaded region.

Pass on C

I-D. You will receive C .

Jane lives C km from work and usually bikes to and from the office. Occasionally she instead rides her bike to work and runs home. She can bike twice as fast as she runs, so doing this extends her overall commute time by 25 minutes.

Let D be the speed (in km/h) at which Jane bikes.

Done!

Team Questions Answer Key

1. 150
2. 30 cm^2
3. 7
4. 52
5. 12:56pm
6. $\frac{5}{18}$
7. $(4, -11)$
8. 5

Pairs Relay Answer Key

- A. 15
- B. 21
- C. 25
- D. 30

Individual Relay Answer Key

- A. 48
- B. 16
- C. 10
- D. 24