

Nova Scotia

Math League

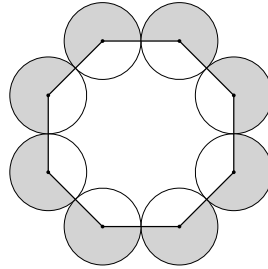
2018–2019

Game Two

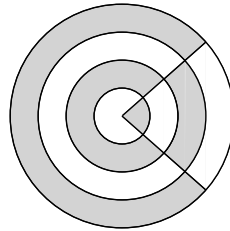
PROBLEMS

Team Questions

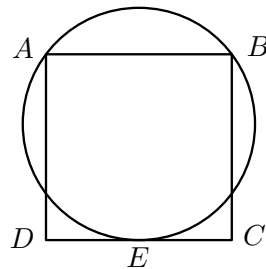
1. In the figure below, each side of the octagon is of length 1, the circles are centred at its vertices, and each circle is tangent to its neighbours. Find the area of the shaded region.



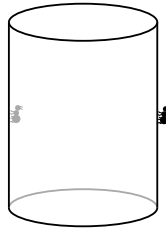
2. Subtracting one from the numerator of a fraction gives $1/4$, while subtracting one from its denominator gives $1/3$. Find the fraction.
3. Let $S(n)$ denote the sum of the digits of n . For example, $S(132) = 1 + 3 + 2 = 6$. Define $a_1, a_2, a_3 \dots$ by setting $a_1 = 1$ and taking $a_n = S(5a_{n-1})$ for $n > 1$. Find a_{2019} .
4. Find the shaded area in the figure below if the indicated angle is 80° and the concentric circles have radii 1, 2, 3, and 4.



5. Vertices A and B of square $ABCD$ lie on a circle, and side CD of the square is tangent to the circle at its midpoint E . If the square has area 4, find the area of the circle.



6. A toilet paper tube has the shape of a hollow cylinder with height 12cm and circumference 10cm. Two ants are on the surface: the first is midway up the outside of the tube and the other is on the inside diametrically opposite the first.



Find the length of the shortest path between the two ants.

(Note: The path must remain on the surface of the tube.)

7. How many subsets of $\{1, 2, 3, 5, 8, 13, 21, 34, 55\}$ have an odd sum? (For example, $\{1, 2, 8\}$ is one such subset, since $1 + 2 + 8 = 11$ is odd.)
8. Sue randomly fills a Tic-Tac-Toe grid with five X's and four O's. Find the probability that there will be at least one line of three X's in a row (horizontal, vertical, or diagonal).

Pairs Relay

P-A. Consider the numbers $1^2, 2^2, 3^2, \dots, 11^2$. Removing the entry A^2 from this list and replacing it with A decreases the average of the list by 10.

Find A .

Pass on A

P-B. You will receive A .

Triangle T has vertices $(5, 1)$, $(A - 2, 5)$ and $(A, 1)$. The midpoints of the sides of T form the vertices of a smaller triangle.

Let B be the area of this smaller triangle.

Pass on B

P-C. You will receive B .

Find the value of C for which the point $(0, C)$ is equidistant from the points $(3, B - 1)$ and $(7, B + 3)$.

Pass on C

P-D. You will receive C .

Let D be the unique digit such that the number DCD is divisible by 7.

Done!

Individual Relay

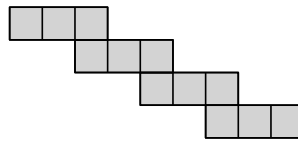
I-A. In the morning Jason bikes to work at 25 km/h, and in the evening he bikes home along the same route at 36 km/h. Jason's journey to work is 11 minutes longer than his journey home.

Let A be the distance (in km) between his home and work.

Pass on A

I-B. You will receive A .

Billy doodles on a sheet of 1×1 grid paper, filling in a number of squares to create the figure shown below.



Billy continues this pattern until he has filled a total of $3A$ squares.

Let B be the **perimeter** of the resulting shape.

Pass on B

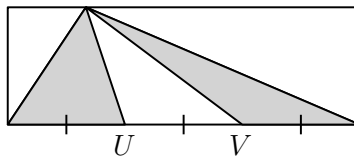
I-C. You will receive B .

Let C be the sum of all integers between 1 and 100 whose last (rightmost) digit is the same as that of B .

Pass on C

I-D. You will receive C .

In the figure below, points U and V divide one side of the rectangle into three equal segments, and the shaded region has area C .



Let D be the area of the unshaded region.

Done!

Team Questions Answer Key

1. $\frac{5\pi}{4}$

2. $\frac{5}{16}$

3. 7

4. $\frac{82\pi}{9}$

5. $\frac{25\pi}{16}$

6. 13

7. 256

8. $\frac{7}{9}$

Pairs Relay Answer Key

A. 11

B. 3

C. 9

D. 5

Individual Relay Answer Key

A. 15

B. 92

C. 470

D. 940