1. Each statement below is True or False.

• The set of solutions of the system consisting of the single equation

$$2x - 3y = 0$$

in the three variables x, y and z is a subspace of $\mathbf{R}^3$.

• Every system of 2 equations in 2 unknowns has a unique solution.

• There is a linear system in 2 variables which is inconsistent.

Choose the correct sequence from the possibilities below.

A. True, True, False.
B. False, False, True.
C. True, False, False.
D. False, True, True.
E. True, False, True.
F. False, True, False.

Please record your answer on the title page.

2. If the vector $v = (2, -1, 0)$ is written as $v = c_1v_1 + c_2v_2 + c_3v_3$, where $\{v_1, v_2, v_3\}$ is the orthonormal basis with

$$v_1 = \frac{\sqrt{2}}{2}(1, 0, 1), \quad v_2 = (0, 1, 0), \quad \text{and } v_3 = \frac{\sqrt{2}}{2}(1, 0, -1),$$

then (c_1, c_2, c_3) is

A. $(\sqrt{2}, 1, \sqrt{2})$
B. $(1, -\sqrt{2}, -\sqrt{2})$
C. $(-\sqrt{2}, 1, \sqrt{2})$
D. $(1, -\sqrt{2}, \sqrt{2})$
E. $(-1, \sqrt{2}, \sqrt{2})$
F. $(\sqrt{2}, -1, \sqrt{2})$

Please record your answer on the title page.

3. Let $v_0 = (1, 0, 1)$ and define

$$U = \{(x, y, z) \in \mathbf{R}^3 \mid x + z = 0\}.$$

i) Show that $v = (x, y, z)$ belongs to U if and only if v is orthogonal to v_0.

ii) Show that $u_1 = (1, 0, 0) - \mathrm{proj}_{v_0}(1, 0, 0)$ and $u_2 = (0, 1, 0) - \mathrm{proj}_{v_0}(0, 1, 0)$ are orthogonal and belong to U.

iii) Give a geometric description of U and show that $\{u_1, u_2\}$ is a basis of U.

iv) If $W = \mathrm{span}\{u_1, u_2, v_0\}$, what is $\dim W$? Is $W = \mathbf{R}^3$?

4.

Consider the vector space $\mathbf{F}[1,2] = \{f \mid f \text{ is a real-valued function defined on } [1,2]\}$ and recall that the zero of $\mathbf{F}[1,2]$ is the function that has the value 0 for all $x \in [1,2]$.

Suppose $f(x) = \frac{1}{x}$ and $g(x) = \frac{1}{x^2}$ and let $W = \operatorname{span}\{f, g\}$.

i) Show that $\{f, g\}$ is linearly independent. What is $\dim W$?

ii) If $h(x) = \frac{2x-3}{x^2}$, show that $h \in W$.

iii) What is the dimension of $\operatorname{span}\{f, g, h\}$?

5. Suppose $\mathcal{B} = \{u, v, w\}$ is a basis of a vector space V.

i) State the two properties of $\mathcal{B}$ that make it a basis of V. What is $\dim V$?

ii) Show that $\mathcal{C} = \{u + 2v, u + 3w, v + w\}$ is linearly independent.

iii) Is $\mathcal{C}$ a basis of V? You must justify your answer, as usual.