UNIVERSITÉ D'OTTAWA • UNIVERSITY OF OTTAWA
Sciences • Science
Mathématiques et statistique
Mathematics and Statistics

MAT 1341B Mid Term Test

February 28, 2002 Duration: 80 minutes.

Instructor: Barry Jessup.

Family Name:______________________

First Name:________________________

Student number:____________________

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2	
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PLEASE READ THESE INSTRUCTIONS CAREFULLY.

1. You have 80 minutes to complete this exam.
2. This is a closed book exam, and no notes of any kind are allowed. **The use of calculators**, cell phones, pagers or any text storage or communication device **is not permitted.**
3. Read each question carefully -you will save yourself time and unnecessary grief later on.
4. Questions 1 and 2 are multiple choice. These questions are worth 2 points each and no part marks will be given. Please record your answers in the space provided above.
5. Questions 3 – 5 require a complete solution, and are worth 6 points each, so spend your time accordingly. **The correct answer requires justification written legibly and logically: you must convince me that you know why your solution is correct. You must answer these questions in the space provided.** Use the backs of pages if necessary.
6. Where it is possible to check your work, do so.
7. Bonne chance! Good luck!

1. Consider the following subsets of $\mathbf{R}^2$:

$U = \{(x, y) \in \mathbf{R}^2 \mid xy = 0\}$, $\quad V = \{(x, y) \in \mathbf{R}^2 \mid x \geq 0, y \geq 0\}$, and $W = \{(x, y) \in \mathbf{R}^2 \mid x+y = 0\}$.

Which of the following statements are true?

I. U is closed under multiplication by scalars.
II. U is closed under addition.
III. V is closed under multiplication by scalars.
IV. V is closed under addition.
V. $\dim W = 2$

A. III & II
B. I & III
C. II & IV
D. III & V
E. I & IV
F. I & V

2. Suppose X is a subspace of $\mathbf{R}^6$, that $X \neq \{0\}$ and that $X \neq \mathbf{R}^6$. Which of the following statements are true?

I. X has a spanning set consisting of 6 vectors.
II. X has a linearly independent subset consisting of 6 vectors.
III. $1 \leq \dim X \leq 5$.
IV. X has a basis that spans $\mathbf{R}^6$.
V. For all vectors u, v, w in X, $au + bv + cw = 0$ implies $a = b = c = 0$.

A. III & II
B. I & III
C. II & IV
D. III & V
E. I & IV
F. I & V

3. Let $W = \{(a, a-b, a+b) \mid a, b \in \mathbf{R}\}$.

a) By any method, show that W is a subspace of $\mathbf{R}^3$.

b) Find a basis of W and give the dimension of W.

c) Give a geometric description of W.

d) Find an equation for W.

4. Let $v_1 = (-1,1,1,1), v_2 = (1,-1,1,1), v_3 = (1,1,-1,1)$, and let W be the subspace of $\mathbf{R}^4$ defined by

$$W = \operatorname{span}\{v_1, v_2, v_3\}.$$

a) Show that $\{v_1, v_2, v_3\}$ is an orthogonal set.

b) Find a basis of W and hence find $\dim W$.

c) The vector $u = (-4, 6, 0, 2)$ belongs to W. Find $c_1, c_2, c_3 \in \mathbf{R}$ such that $u = c_1v_1 + c_2v_2 + c_3v_3$.

d) Show that $\{v_1 + 2v_2, v_1 + 3v_3, v_2 + v_3\}$ is a basis of W.

e) Assuming that $v_4 = (1, 0, 0, 0)$ does not belong to W (you do **not** have to check this), explain why $\{v_1, v_2, v_3, v_4\}$ is a basis of $\mathbf{R}^4$.

5. In each case, give an explicit example of:

a) An inconsistent linear system of 2 equations in 2 unknowns. Sketch the graphs of the equations of your system to illustrate your example.

b) A linear system of 2 equations in 3 unknowns with at least 2 different solutions. Give two different solutions.

c) A linear system of 4 equations in 4 unknowns with a unique solution. Give the unique solution.