MAT 1341, INTRODUCTION TO LINEAR ALGEBRA, WINTER 2003

Midterm, March 7

Prof. P. Selinger

FAMILY NAME: ________________ **FIRST NAME:** ________________ **ID:** ________________

Question:	1	2	3	4	5	6	7	8	Total
Possible Points	2	2	2	2	3	3	4	4	22
Actual Points:									

PLEASE READ THESE INSTRUCTIONS CAREFULLY.

1. You have 80 minutes to complete this test.
2. This is a closed book test, and no notes of any kind are allowed. The use of calculators is neither required nor permitted.
3. The test has 8 questions.

   Questions 1–6 are multiple choice or short answer questions. Make sure you check your answer carefully, as there will be no part marks given.

   Questions 7–8 require a detailed answer. Please write legibly and reason carefully. You can write on the back of pages if necessary.

Question 1. Under what condition can a point (a, b, c) be written as a linear combination of $(1, 2, 0)$ and $(1, 1, 1)$?

☐ $3a - b - 2c = 0.$

☐ $a + b - 2c = 0.$

☐ $2a - b - c = 0.$

☐ $2a - b + 2c = 0.$

☐ $a - b = 0.$

☐ $a - 3b + 2c = 0.$

Question 2. Suppose that a given matrix A satisfies $A^2 - 2A - I = 0$. Give a formula for A^{-1}:

☐ $A^{-1} = 2A + I.$

☐ $A^{-1} = A + I.$

☐ $A^{-1} = A - 2I.$

☐ $A^{-1} = 2A - I.$

☐ $A^{-1} = A + 2I.$

☐ $A^{-1} = A - I.$

Question 3. In the vector space $\mathbf{M}_{2,2}$ of real 2×2-matrices, express M as a linear combination of the matrices A, B, and C, where

$$M = \begin{pmatrix} 4 & 7 \\ 7 & 9 \end{pmatrix}, \qquad A = \begin{pmatrix} 1 & 1 \\ 4 & 5 \end{pmatrix}, \qquad B = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}, \qquad C = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix},$$

Answer: $M = \underline{\qquad} A + \underline{\qquad} B + \underline{\qquad} C$

Question 4. Write the polynomial $p(t) = t^2 - 2t + 1$ as a linear combination of the three polynomials $q_1(t) = t^2 - 1$, $q_2(t) = t^2 + t$ and $q_3(t) = t^2 + t + 1$.

Answer: $p = \underline{\qquad} q_1 + \underline{\qquad} q_2 + \underline{\qquad} q_3$

Question 5. Determine whether or not W is a subspace of $\mathbb{R}^3$ where W consists of all vectors (a, b, c) in $\mathbb{R}^3$ such that:

(a)	$b = a^2$	☐ yes	☐ no
(b)	$a = 2b = 3c$	☐ yes	☐ no
(c)	$a = 3b$	☐ yes	☐ no
(d)	$ab = 0$	☐ yes	☐ no
(e)	$a \leqslant b \leqslant c$	☐ yes	☐ no
(f)	$a + b + c = 0$	☐ yes	☐ no

Question 6. Let V be a vector space over a field K. Which of the following statements are always valid:

(a) Every subset of V is a subspace of V ☐ true ☐ false

(b) Every subspace of V is a subset of V ☐ true ☐ false

(c) $\{0\}$ is a subspace of V ☐ true ☐ false

(d) Let $u, v \in V$ be vectors, and let W be a subspace of V. If W contains the vectors u and v, then W also contains the sum $u + v$. ☐ true ☐ false

(e) Let $u, v \in V$ be vectors, and let W be a subspace of V. If W contains the sum $u + v$, then W also contains u and v. ☐ true ☐ false

Question 7. Let W be a subset of a vector space V over a field K.

(a) Write down the three conditions which W must satisfy in order to be a subspace of V:

(1)

(2)

(3)

(b) Show that $W = \{(x, y, z) \in \mathbb{R}^3 \mid x + y \geqslant z\}$ is not a subspace of $\mathbb{R}^3$ by showing, in a specific example, that one of the above three conditions is violated.

Question 8. (a) Let $V = \mathbf{P}(t)$ be the vector space of polynomials in the variable t, with real coefficients. Let $W = \{p(t) \in V \mid p(2) = p(-2)\}$. Show that W is a subspace of V.

(b) Recall that an $n \times n$-matrix A is called *symmetric* if $A = A^T$. Prove that the set of all symmetric $n \times n$-matrices is a subspace of $\mathbf{M}_{n,n}$.