

**MAT 3343, APPLIED ALGEBRA, FALL 2003**

**Problem Set 3, due Oct 10, 2003**

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**Problem 1.** My public RSA key is  $\langle N, e \rangle = \langle 11639, 65 \rangle$ . Send me an encrypted message. Encode each 2-letter pair as a 4-digit decimal number with Space=00, A=01, B=02, etc. For instance, the message "Hello" would be encoded in plaintext as the sequence of numbers 0805 1212 1500. (Calculators are allowed for this problem!)

**Problem 2.** Suppose that  $N = pq$  is the product of two distinct primes, possibly very large. Suppose  $p, q$  are unknown, but  $N$  is known. Further, assume given an element  $x \in \mathbb{Z}_N$  such that  $x^2 = x$ , but  $x \neq 0, 1$ . Show that from this information, one can efficiently compute  $p$  and  $q$ .

**Problem 3.** State and prove the generalized Chinese Remainder Theorem (see Exercise 1.1, Handout 3 or Problem 34, p.114)

**Problem 4.** Suppose  $N = pqr$  is the product of three distinct odd primes.

- (a) How many square roots of unity are there in  $\mathbb{Z}_N$ ?
- (b) Show that the set of such square roots can be computed efficiently if  $p, q, r$  are known.
- (c) Compute the set of square roots of unity for  $N = pqr$  where  $p = 7, q = 11$ , and  $r = 13$ .
- (d) Suppose  $p, q, r$  are not known, but some square root of unity  $x \in \mathbb{Z}_N$  is known such that  $x \neq \pm 1$ . What information, if any, can be gained about the prime factorization of  $N$ ? (Hint: use a similar idea as in the first paragraph of the proof of Theorem 3.2, Handout 3).

**Problem 5.** (a) Use the Fermat pseudoprime test (Algorithm 1.3, Handout 4) to show that the number 119 is not prime. In particular, find some  $b$  such that the number 119 fails the Fermat pseudoprime test at base  $b$ .

- (b) Use the Miller-Rabin primality test (Algorithm 3.4, Handout 4) to show that 561 is not prime. (In particular, find some  $b$  such that the number 561 fails the strong pseudoprime test at base  $b$ ).