

Answers to Problem Set 2:

Handout 1, 11, 21, 31, 41, 51 (using Prawitz style natural deduction),
12, 22, 32, 42, 52 (using analytic tableaux)

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Problem 11

$$(a) \frac{A \wedge (A \vee B)}{A} (\wedge E) \qquad (b) \frac{A \quad \frac{A}{A \vee B} (\vee I)}{A \wedge (A \vee B)} (\wedge I)$$

Problem 21

$$\frac{\frac{\frac{[A \wedge B]_1}{B} (\wedge E) \quad \frac{A \wedge \neg B}{\neg B} (\wedge E)}{\perp} (\neg E)}{\neg(A \wedge B)} (\neg I_1)$$

Problem 31

$$(a) \frac{\frac{\frac{[A]_1}{A \vee B} (\vee I) \quad \frac{A \vee B \rightarrow C}{A \rightarrow C} (\rightarrow I_1)}{A \rightarrow C} (\rightarrow E) \quad \frac{\frac{[B]_2}{A \vee B} (\vee I) \quad \frac{A \vee B \rightarrow C}{B \rightarrow C} (\rightarrow I_2)}{B \rightarrow C} (\rightarrow E)}{(A \rightarrow C) \wedge (B \rightarrow C)} (\wedge I)$$

$$(b) \frac{[A \vee B]_1 \quad \frac{\frac{[A]_2}{A \rightarrow C} (\rightarrow E) \quad \frac{[B]_2}{B \rightarrow C} (\rightarrow E)}{C} (\vee E_{2,3})}{\frac{C}{(A \vee B) \rightarrow C} (\rightarrow I_1)}$$

Problem 41

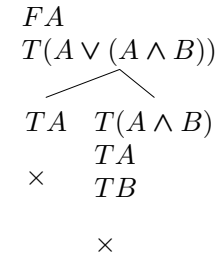
$$\frac{[\neg(A \vee \neg A)]_1 \quad \frac{\frac{[A]_2}{A \vee \neg A} (\vee I)}{\perp} (\neg I_2)}{\frac{\perp}{A \vee \neg A} (\neg E)} (\neg E)$$

Problem 51

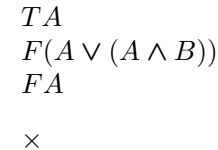
$$\frac{[\neg((A \rightarrow B) \vee (B \rightarrow A))]_1 \quad \frac{\frac{[A]_2}{B \rightarrow A} (\rightarrow I_3)}{(A \rightarrow B) \vee (B \rightarrow A)} (\vee E)}{\frac{\perp}{B} (\perp E)} (\neg E)$$

$$\frac{\frac{\frac{\perp}{A \rightarrow B} (\rightarrow I_2)}{(A \rightarrow B) \vee (B \rightarrow A)} (\vee E)}{\frac{\perp}{(A \rightarrow B) \vee (B \rightarrow A)} (\neg E)} (\neg E)$$

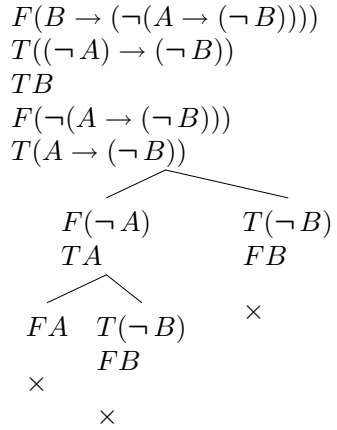
Problem 12 (a) $A \vee (A \wedge B) \vdash A$



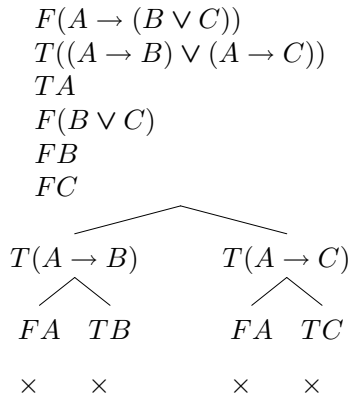
(b) $A \vdash A \vee (A \wedge B)$



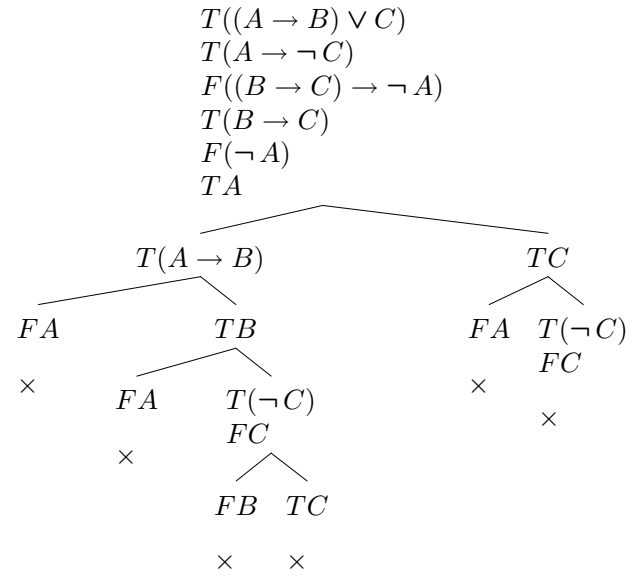
Problem 22



Problem 32



Problem 42



Problem 52

