MATH/CSCI 2112, DISCRETE STRUCTURES I, FALL 2005

Handout 1: Translation and Statement Algebra

Each of problems 1–3 below gives a specific argument, which may or may not be valid. Translate the arguments into propositional logic using the letters shown. Then check whether or not the argument is valid, using one of the methods shown in class (truth tables, tableaux). If the argument is not valid, give a specific counterexample.

Please refer to the last page of this handout for a list of English phrases that are commonly used for logical connectives.

Problem 1. If the lion does not hide, then the hunt ends soon. If the hunt ends soon, then either the lion is killed or the hunter is eaten. The lion is not killed. Therefore the hunter is eaten unless the lion hides.

Use the following abbreviations in your translations:

L: the lion hides
H: the hunt ends soon
K: the lion is killed
E: the hunter is eaten

Problem 2. If the dog is barking, and the dog is in the house, then someone is at the front door. If someone is at the front door and the dog is not in the house, then the dog is barking. The dog only barks if nobody is at the front door. Therefore, if the dog is barking, the dog is in the house.

B: the dog is barking
H: the dog is in the house
F: someone is at the front door

Problem 3. The following argument refers to this fragment of a Java or C program:

```
/* label 1 */
 if (i != 0) {
   j = 0;
 }
/* label 2 */
 i = i + j;
 j = i - j;
/* label 3 */
```

Argument:

If not $i = 0$ at label 1, then $j = 0$ at label 2. Also, $i = 0$ at label 2 if and only if $i = 0$ at label 1. If $i = 0$ at label 2, then $j = 0$ at label 3. If $j = 0$ at label 2, then $j = i$ at label 3. Therefore, at label 3, either $j = 0$ or $j = i$.

Use the following abbreviations in your translations:

A: $i = 0$ at label 1
B: $i = 0$ at label 2
C: $j = 0$ at label 2
D: $j = 0$ at label 3
E: $j = i$ at label 3

Problem 4. Translate each of the following sentences into propositional logic, using the following letters for atomic formulas. If there is more than one possible translation, choose the most likely one and justify your choice.

T: the government raises taxes
R: there is a recession
E: exports exceed imports
D: there is a deficit
N: trade increases

1. The government does not raise taxes unless there is a recession or a deficit.
2. If trade increases, the goverment does not raise taxes.
3. There is a recession although exports exceed imports.
4. Trade increases if exports do not exceed imports.
5. The government raises taxes when and only when there is a recession or exports exceed imports.
6. There is no deficit, provided that trade increases.
7. Provided that there is no deficit, trade increases.
8. The government raises taxes, otherwise there is a deficit.
9. Exports exceed imports, however there is still a recession.
10. Whenever trade increases, the government raises taxes.
11. So long as trade increases, there is no recession.
12. In order for trade to increase, it is necessary that the government does not raise taxes.
13. In order for trade to increase, it is sufficient that the government does not raise taxes.

Problem 5. Translate the following arguments into propositional logic, and decide whether each argument is valid or not. Use the same propositional letters as in the previous problem.

1. If the government raises taxes, trade increases. If trade increases or exports exceed imports, there is a deficit. Therefore, there is a deficit unless the government does not raise taxes.
2. If there is a deficit, trade does not increase. Increasing trade is necessary for exports to exceed imports. Therefore, whenever exports exceed imports, there is a deficit.
3. The government raises taxes since trade increases. The government does not raise taxes unless there is a deficit. Therefore, there is a deficit.

Laws of Statement Algebra

Laws for $\rightarrow$ and $\leftrightarrow$

$p \rightarrow q$	$\equiv$	$\sim p \vee q$
$p \leftrightarrow q$	$\equiv$	$(p \wedge q) \vee (\sim p \wedge \sim q)$
$p \leftrightarrow q$	$\equiv$	$(p \rightarrow q) \wedge (q \rightarrow p)$

Laws for $\sim$, $\wedge$ and $\vee$

$p \vee \sim p$	$\equiv$	**T**	Excluded middle
$p \wedge \sim p$	$\equiv$	**F**	Contradiction
$p \vee \mathbf{F}$	$\equiv$	p	Identity for $\vee$
$p \wedge \mathbf{T}$	$\equiv$	p	Identity for $\wedge$
$p \vee \mathbf{T}$	$\equiv$	**T**	Domination for $\vee$
$p \wedge \mathbf{F}$	$\equiv$	**F**	Domination for $\wedge$
$p \vee p$	$\equiv$	p	Idempotence of $\vee$
$p \wedge p$	$\equiv$	p	Idempotence of $\wedge$
$\sim\sim p$	$\equiv$	p	Double negation
$p \vee q$	$\equiv$	$q \vee p$	Commutativity of $\vee$
$p \wedge q$	$\equiv$	$q \wedge p$	Commutativity of $\wedge$
$(p \vee q) \vee r$	$\equiv$	$p \vee (q \vee r)$	Associativity of $\vee$
$(p \wedge q) \wedge r$	$\equiv$	$p \wedge (q \wedge r)$	Associativity of $\wedge$
$p \vee (q \wedge r)$	$\equiv$	$(p \vee q) \wedge (p \vee r)$	Distributivity of $\vee$ over $\wedge$
$p \wedge (q \vee r)$	$\equiv$	$(p \wedge q) \vee (p \wedge r)$	Distributivity of $\wedge$ over $\vee$
$\sim(p \wedge q)$	$\equiv$	$\sim p \vee \sim q$	De Morgan's law
$\sim(p \vee q)$	$\equiv$	$\sim p \wedge \sim q$	De Morgan's law

Example. (Proof by algebraic manipulations)

Here is a proof of $X \vee (X \wedge Y) \equiv X$:

$$\begin{aligned}
X \vee (X \wedge Y) &\equiv (X \wedge \mathbf{T}) \vee (X \wedge Y) && \text{(Identity for } \wedge\text{)} \\
&\equiv X \wedge (\mathbf{T} \vee Y) && \text{(Distributivity of } \wedge \text{ over } \vee\text{)} \\
&\equiv X \wedge \mathbf{T} && \text{(Domination for } \vee\text{)} \\
&\equiv X && \text{(Identity for } \wedge\text{)}
\end{aligned}$$

Problem 6. The following is a truth table for the 16 possible propositions with propositional variables p and q. (a) Find formulas for propositions $f_0, \ldots, f_7$, using only the connectives "$\sim$" and "$\vee$". (b) Find formulas for propositions $f_8, \ldots, f_{15}$, using only the connectives "$\sim$" and "$\rightarrow$".

p	q	f_0	f_1	f_2	f_3	f_4	f_5	f_6	f_7	f_8	f_9	f_{10}	f_{11}	f_{12}	f_{13}	f_{14}	f_{15}
T	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**F**	**F**	**F**	**F**	**F**	**F**	**F**	**F**
T	**F**	**T**	**T**	**T**	**T**	**F**	**F**	**F**	**F**	**T**	**T**	**T**	**T**	**F**	**F**	**F**	**F**
F	**T**	**T**	**T**	**F**	**F**	**T**	**T**	**F**	**F**	**T**	**T**	**F**	**F**	**T**	**T**	**F**	**F**
F	**F**	**T**	**F**	**T**	**F**	**T**	**F**	**T**	**F**	**T**	**F**	**T**	**F**	**T**	**F**	**T**	**F**

Some English phrases for propositional connectives:

$\sim p$:

- not p
- it is false that p
- p is false
- p is not true
- p doesn't happen
- p fails

$p \wedge q$:

- p and q
- both p and q
- p but q
- not only p, but also q
- p, and also q
- p although q
- p even though q
- p, however q
- p, whereas q
- p besides q
- p, nevertheless q
- p, nonetheless q
- p in spite of the fact that q
- despite p, q
- p, while q
- while p, q
- p since q
- p together with q
- p as well as q
- the conjunction of p and q

$p \vee q$:

- p or q
- either p or q
- either p or q or both
- p or else q
- p, or alternatively q
- p, otherwise q
- p unless q

$p \longrightarrow q$:

- p implies q
- if p then q
- if p, q
- q if p
- p only if q
- only if q, p
- q follows from p
- given that p, q follows
- provided that p, q
- q, provided that p
- whenever p, q
- in case p, q
- q is necessary for p
- q is a necessary condition for p
- p is a sufficient condition for q

$p \leftrightarrow q$:

- p if and only if q
- p is equivalent to q
- p exactly in case q
- p just in case q
- p is a necessary and sufficient condition for q

$p \oplus q$:

- p or q, but not both
- exactly one of p and q is true

neither p nor q:

- $\sim(p \vee q)$
- $\sim p \wedge \sim q$

if p then q, else r:

- $(p \longrightarrow q) \wedge (\sim p \longrightarrow r)$
- $(p \wedge q) \vee (\sim p \wedge r)$