# MATHEMATICS 1115 2007 DECEMBER 13 8:30AM

**SECTION ________ NAME (PRINTED) ______________________________**

**Student Number ______________ SIGNATURE ___________________________**

***Only calculators without memory or graphics are allowed and no other electronic devices are allowed.*** **Each multiple choice question is worth 1 mark. All work is to be shown on the attached question paper in the space provided and *the answer selected must be* X*'d on this answer sheet.* Both the question paper and the answer sheet, *both* with completed identification sections, must be handed in.**

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# MATHEMATICS 1115 2007 DECEMBER 13 8:30

# EXAMINATION

**SECTION __________ NAME (PRINTED) ________________________________**

**Student Number _______________ SIGNATURE ___________________________**

***Only calculators without memory or graphics are allowed and no other electronic devices are allowed.*** **Each multiple choice question is worth 1 mark. All work is to be shown on this question paper in the space provided and *the answer selected must be* X*'d on the attached answer sheet.* Both the question paper and the answer sheet, *both* with completed identification sections, must be handed in.**

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$a_{\overline{n}|r}$ is the *present* value of an ordinary annuity of \$1 per period for $n$ periods at the interest rate of $r$ per period. It is given by

$$a_{\overline{n}|r} = \frac{1-(1+r)^{-n}}{r}$$

$s_{\overline{n}|r}$ is the *future* value of an ordinary annuity of \$1 per period for $n$ periods at the interest rate of $r$ per period. It is given by

$$s_{\overline{n}|r} = \frac{(1+r)^{n}-1}{r}$$

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1. If a principal is invested at annual rate of 5% compounded annually then the number of years it will take to double is:

(A) 20 (B) $\frac{\ln 2}{\ln 1.05}$ (C) 2 (D) $\frac{\ln 20}{\ln 1.05}$ (E) none of these

2. A debt of \$550 due in four years and \$550 due in five years is to be repaid by a single payment now. Interest is at the rate of 10% compounded quarterly. The amount, in dollars, of the single payment is:

(A) 1100 (B) $550(1.025)^{16}+550(1.025)^{20}$ (C) $550(1.10)^{-4}+550(1.10)^{-5}$
(D) $550(1.10)^4 + 550(1.10)^5$ (E) $550(1.025)^{-16} + 550(1.025)^{-20}$

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3. If \$1000 is invested at an annual rate of 3% compounded continuously then at the end of eight years the compounded amount, in dollars, is:

(A) $1000(1.03)^8$ (B) $1000e^{0.24}$ (C) $1000(1.03)^{-8}$ (D) $1000a_{\overline{8}|0.03}$ (E) $1000s_{\overline{8}|0.03}$

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4. A machine is purchased for \$3000 and payments of \$250 at the end of every six months for six years. Interest is at 8% compounded semiannually. The corresponding cash price, in dollars, is:

(A) $3000-250a_{\overline{12}|0.08}$ (B) $3000+250s_{\overline{12}|0.08}$ (C) $3000+250s_{\overline{12}|0.04}$ (D) $3000+250a_{\overline{12}|0.04}$
(E) $3000-250a_{\overline{12}|0.04}$

5. A loan of \$2000 is being amortized over 48 months at an interest rate of 12% compounded monthly. The monthly payment, in dollars is:

(A) $\dfrac{2000}{a_{\overline{48}|0.01}}$ (B) $\dfrac{a_{\overline{48}|0.01}}{2000}$ (C) $\dfrac{2000}{s_{\overline{48}|0.01}}$ (D) $\dfrac{2000}{a_{\overline{4}|0.12}}$ (E) $\dfrac{s_{\overline{48}|0.01}}{2000}$

The next *TWO* questions concern the system $\begin{cases} x \quad\quad\quad + \;3z = -1 \\ 3x + 2y + 11z = 1 \\ x + y + 4z = 1 \\ 2x - 3y + 3z = -8 \end{cases}$.

6. The augmented matrix of the system reduces to:

(A) $\left[\begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$ (B) $\left[\begin{array}{ccc|c} 1 & 0 & 3 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$ (C) $\left[\begin{array}{ccc|c} 1 & 0 & 3 & -1 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$

(D) $\left[\begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]$ (E) $\left[\begin{array}{ccc|c} 1 & 0 & 3 & -1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right]$

7. The solutions of the system are:

(A) no solutions (B) $x = -1$, $y = 2$, $z = 0$
(C) $x = -1 - 3r$, $y = 2 - r$, $z = r$, for $r$ any real number
(D) $x = -1 - 3r$, $y = r$, $z = s$, for $r$ and $s$ any real numbers
(E) $x = -1 - 3r$, $y = 2 - r$, $z = 1$, for $r$ any real number

8. A simple economy with 3 sectors: water ($W$), electric power ($E$), and agriculture ($A$), has a Leontieff coefficient matrix given by $\begin{array}{c c} & \begin{array}{ccc} W & E & A \end{array} \\ \begin{array}{c} W \\ E \\ A \end{array} & \begin{pmatrix} \frac{1}{10} & \frac{1}{2} & \frac{1}{5} \\ \frac{1}{10} & \frac{1}{10} & \frac{2}{5} \\ \frac{3}{10} & \frac{1}{5} & \frac{1}{5} \end{pmatrix} \end{array}$.
There is an (external) demand for 300 units of water, 200 units of electric power, and 400 units of agriculture. The production required to satisfy the external demand is found by solving:

A) $\begin{cases} \frac{9}{10}w - \frac{1}{2}e - \frac{1}{5}a = 300 \\ -\frac{1}{10}w + \frac{9}{10}e - \frac{2}{5}a = 200 \\ -\frac{3}{10}w - \frac{1}{5}e + \frac{4}{5}a = 400 \end{cases}$ (B) $\begin{cases} \frac{1}{10}w + \frac{1}{2}e + \frac{1}{5}a = 300 \\ \frac{1}{10}w + \frac{1}{10}e + \frac{2}{5}a = 200 \\ \frac{3}{10}w + \frac{1}{5}e + \frac{1}{5}a = 400 \end{cases}$

(C) $\begin{cases} \frac{1}{10}w + \frac{1}{10}e + \frac{3}{10}a = 300 \\ \frac{1}{2}w + \frac{1}{10}e + \frac{1}{5}a = 200 \\ \frac{1}{5}w + \frac{2}{5}e + \frac{1}{5}a = 400 \end{cases}$ (D) $\begin{cases} \frac{9}{10}w - \frac{1}{10}e - \frac{3}{10}a = 300 \\ -\frac{1}{2}w + \frac{9}{10}e - \frac{2}{5}a = 200 \\ -\frac{1}{5}w - \frac{2}{5}e + \frac{4}{5}a = 400 \end{cases}$ (E) $\begin{cases} w = 300 \\ e = 200 \\ a = 400 \end{cases}$

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The next *FOUR* questions involve the problem:

$$\text{Maximize} \qquad P = 4x_1 + 0x_2 - x_3$$

$$\text{Subject to} \quad \begin{cases} x_1 + x_2 + x_3 \leq 6 \\ x_1 - x_2 + x_3 \leq 10 \\ x_1 - x_2 - x_3 \leq 4 \\ x_1, x_2, x_3 \geq 0 \end{cases}$$

for which it can be shown that the *second* simplex table is:

$$\begin{array}{c} B \\ s_1 \\ s_2 \\ x_1 \\ P \end{array}
\begin{array}{c} \begin{array}{cccccccccc} x_1 & x_2 & x_3 & s_1 & s_2 & s_3 & P & & R \end{array} \\
\left[\begin{array}{ccccccc|c} 0 & 2 & 2 & 1 & 0 & -1 & 0 & 2 \\ 0 & 0 & 2 & 0 & 1 & -1 & 0 & 6 \\ 1 & -1 & -1 & 0 & 0 & 1 & 0 & 4 \\ \hline 0 & -4 & -3 & 0 & 0 & 4 & 1 & 16 \end{array}\right] \end{array}$$

9. This table tells us that:

   (A) $P = 0$ at $(4, 0, 0)$ (B) $P = -16$ at $(2, 6, 4)$ (C) $P = 16$ at $(2, 6, 4)$
   (D) $P = 16$ at $(4, 0, 0)$ (E) $P = -16$ at $(4, 0, 0)$

10. The pivot column in the table shown is the one labelled by:

    (A) $x_1$ (B) $x_2$ (C) $x_3$ (D) $s_3$ (E) $P$

11. The pivot row in the table shown is the one labelled by:

    (A) $B$ (B) $P$ (C) $x_1$ (D) $s_2$ (E) $s_1$

12. The $P$ row of the *next* table is:

    (A) $\left[\begin{array}{ccccccc|c} 0 & 0 & 1 & 2 & 0 & 2 & 1 & 18 \end{array}\right]$ (B) $\left[\begin{array}{ccccccc|c} 0 & 0 & -1 & 2 & 0 & 2 & 1 & 20 \end{array}\right]$
    (C) $\left[\begin{array}{ccccccc|c} 0 & 0 & 1 & 2 & 0 & 2 & 1 & 2 \end{array}\right]$ (D) $\left[\begin{array}{ccccccc|c} 0 & 0 & 1 & 2 & 0 & 2 & 1 & 0 \end{array}\right]$ (E) $\left[\begin{array}{ccccccc|c} 0 & 0 & 1 & 2 & 0 & 2 & 1 & 20 \end{array}\right]$

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13. For a function $C$, one can find "$\min C$" as:

(A) $-\max(-C)$ (B) $\max(-C)$ (C) $-\max(C)$ (D) $-\max(C^{-1})$ (E) $(\max(C^{-1}))^{-1}$

14 A coin is tossed five times. The number of possible results is:

(A) ${}_5P_2$ (B) $5 \cdot 2$ (C) $5^2$ (D) $2^5$ (E) ${}_5C_2$

15. From a deck of 52 playing cards, the number of 5-card hands comprised solely of red cards is:

(A) ${}_{26}C_5$ (B) ${}_{52}C_5$ (C) ${}_{26}P_5$ (D) $\frac{26}{5}$ (E) $26^5$

16. If $P(E) = \frac{1}{2}$, $P(E \cup F) = \frac{13}{20}$, and $P(E \cap F) = \frac{1}{10}$ then $P(F) =$

(A) $\frac{1}{5}$ (B) $\frac{3}{20}$ (C) $\frac{1}{4}$ (D) $\frac{3}{10}$ (E) $\frac{1}{2}$

17. If $P(E) = \frac{3}{5}$, $P(F) = \frac{3}{10}$, and $P(E \cup F) = \frac{7}{10}$ then $P(E|F) =$

(A) $\frac{1}{3}$ (B) $\frac{3}{2}$ (C) $\frac{2}{3}$ (D) $\frac{2}{5}$ (E) $\frac{9}{10}$

18. Out of 3000 tires in a warehouse, 2000 are domestic and 1000 are imported. Among the domestic tires 2/5 are all-season and among the imported tires 1/10 are all-season. A tire is selected at random and found to be all-season. The probability that it is imported is:

(A) $\frac{4}{15}$ (B) 9% (C) 10% (D) $\frac{1}{8}$ (E) $\frac{1}{9}$

19. You and a friend play a game. You toss three fair coins. If three heads or three tails turn up, your friend pays you \$10. If either one or two heads turn up, you pay your friend \$6. Your expected winnings per game, in dollars, are:

(A) $2$ (B) $-2$ (C) $10$ (D) $4$ (E) $8$

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20. Each question in a six-question multiple-choice test has four choices, only one of which is correct. A student guesses at all six questions. The probability that she gets exactly three correct is:

(A) $\dfrac{3}{6}$ (B) $120 \cdot \dfrac{3^3}{4^6}$ (C) $\dfrac{1}{4^3}$ (D) $20 \cdot \dfrac{3^3}{4^6}$ (E) $\dfrac{3^3}{4^6}$

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21. $\displaystyle \lim_{h \to 0} \frac{((x+h)^2 + (x+h) + 1) - (x^2 + x + 1)}{h} =$

(A) $2x+1$ (B) $0$ (C) $\infty$ (D) $-\infty$ (E) $2x+2$

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22. If $f(x) = \frac{6}{7}^{2/3}$ then $f'(x) =$

(A) $(2/3)(6/7)^{-1/3}$ (B) $(2/3)(6/7)^{1/3}$ (C) $(-1/3)(6/7)^{-1/3}$ (D) 0 (E) 1

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The next *TWO* questions concern $c = 5000 + 6q$, where $c$ is the cost of producing $q$ units of a product.

23. The marginal cost function is:

(A) $\dfrac{5000 + 6q}{q}$ (B) 6 (C) 5000 (D) $6q$ (E) $5000q + 3q^2$

24. When 3 units are produced the marginal cost is:

(A) 5000 (B) 18 (C) $\dfrac{5018}{3}$ (D) $15,027$ (E) 6

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25. If $Q(x) = (3 + x)(5x^2 - 2)$ then $Q'(x) =$

(A) $10x$ (B) $3(5x^2 - 2) + (3 + x)(5x - 2)$ (C) $(1)(5x^2 - 2) + (3 + x)(10x)$
(D) $(3)(10)$ (E) $(3 + x)(5x^2 - 2) + (3 + x)(10x)$

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26. If $H(x) = \dfrac{-5x}{5-x}$ then $H'(x) =$

(A) $\dfrac{(5-x)(-5)-(-5x)(-1)}{(5-x)^2}$ (B) $\dfrac{-5}{-1}$ (C) $-5$ (D) $\dfrac{(-5x)(-1)-(5-x)(-5)}{(5-x)^2}$
(E) none of these

27. If $y = (x^2-4)^4$ then $\dfrac{dy}{dx} =$

(A) $4(x^2-4)^3$ (B) $(2x)^4$ (C) $4(2x)^3$ (D) $4(x^2)^3(2x)$ (E) $4(x^2-4)^3(2x)$

28. If $y = \sqrt{3x^2-7}$ then $\dfrac{dy}{dx} =$

(A) $\dfrac{1}{2\sqrt{3x^2-7}}$ (B) $\dfrac{6x}{2\sqrt{3x^2-7}}$ (C) $\dfrac{6x}{\sqrt{3x^2-7}}$ (D) $\dfrac{6x}{(3x^2-7)^{3/2}}$ (E) $\dfrac{1}{2\sqrt{6x}}$

29. If $y = e^{2x^2+5}$ then $\dfrac{dy}{dx} =$

(A) $5xe^{2x^2+4}$ (B) $e^{4x}$ (C) $(2x^2+5)e^{2x^2+4}$ (D) $4xe^{2x^2+5}$ (E) $4xe^{4x}$

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30. If $y = 3x^4e^{-x}$ then $\dfrac{dy}{dx} =$

(A) $12x^3e^{-x} - 3x^4e^{-x}$ (B) $12x^3e^{-x} + 3x^4(-x)e^{-x-1}$ (C) $-12x^3e^{-x}$ (D) $12x^3e^{-x} + 3x^4e^{-x}$
(E) $3x^4e^{-x}$

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31. If $y = \ln(5x-6)$ then $\dfrac{dy}{dx} =$

(A) $\dfrac{1}{\ln(5x-6)}$ (B) $\dfrac{1}{5x-6}$ (C) $\dfrac{5}{\ln(5x-6)}$ (D) $\dfrac{5}{5x-6}$ (E) $\dfrac{1}{5}$

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32. If $y = \dfrac{x^2}{\ln x}$ then $\dfrac{dy}{dx} =$

(A) $\dfrac{2x}{\frac{1}{x}}$ (B) $\dfrac{(\ln x)(2x) - (x^2)\left(\frac{1}{x}\right)}{(\ln x)^2}$ (C) $\dfrac{2x}{x}$ (D) $\dfrac{(x^2)\left(\frac{1}{x}\right) - (\ln x)(2x)}{(\ln x)^2}$ (E) $\dfrac{x^2}{\frac{1}{x}}$

The demand equation for a certain product is $q = \sqrt{2500 - p^2}$. The next *THREE* questions refer to this situation.

33. The point elasticity of demand, $\eta$, is:

(A) $\dfrac{q^2}{p^2}$ (B) $\dfrac{p^2}{q^2}$ (C) $-\dfrac{p^2}{q}$ (D) $-\dfrac{p^2}{q^2}$ (E) $-\dfrac{q^2}{p^2}$

34. The value of $\eta$ when $p = 30$ is:

(A) $-\dfrac{16}{9}$ (B) $\dfrac{16}{9}$ (C) $-\dfrac{3}{4}$ (D) $-\dfrac{9}{16}$ (E) $\dfrac{9}{16}$

35. If the current price of 30 is decreased by 5% then: demand will increase

by:

(A) $\frac{45}{16}\%$ (B) $-\frac{45}{16}\%$ (C) $\frac{16}{45}\%$ (D) $\frac{80}{9}\%$ (E) $-\frac{80}{9}\%$

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The next *TWO* questions concern the demand equation

$$p = 400 - 40q - q^2$$

where $p$ is price (in dollars) and $q$ is quantity demanded.

36. The marginal revenue function is given by:

(A) $400-80q-3q^2$ (B) $400-40q-q^2$ (C) $400q-40q^2-q^3$ (D) $40-2q$ (E) $40q-2q^2$

37. When $q = 4$ the rate at which marginal revenue is changing, in dollars per unit per unit, is:

(A) $-32$ (B) $32$ (C) $-104$ (D) $104$ (E) $-304$

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38. The function $f(x) = -2x^2 - 6x + 5$ has, on the interval $[-3, 2]$, an absolute minimum value of:

(A) $-\frac{3}{2}$ (B) $5$ (C) $-15$ (D) $\frac{19}{2}$ (E) $-27$

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The next *FIVE* questions are concerned with the function

$$f(x) = 5x^{2/3} - x^{5/3}$$

39. The function $f$ has intervals of increase and intervals of decrease as follows:

(A) increasing on $(-\infty, \infty)$ (B) increasing on $(-\infty, 0)$ and $(2, \infty)$; decreasing on $(0, 2)$
(C) increasing on $(0, 2)$; decreasing on $(-\infty, 0)$ and $(2, \infty)$ (B) decreasing on $(-\infty, \infty)$
(E) decreasing on $(-\infty, 0)$; increasing on $(0, \infty)$

40. The relative extrema of the function $f$ are as follows:

(A) relative maximum at $x = 0$; relative minimum at $x = 2$ (B) no relative extrema
(C) relative maximum at $x = 2$ (D) relative minimum at $x = 2$
(E) relative minimum at $x = 0$; relative maximum at $x = 2$

41. Concavity of the function $f$ is described as follows:

(A) concave up on $(-\infty, \infty)$ (B) concave up on $(-\infty, 0)$; concave down on $(0, \infty)$
(C) concave down on $(-\infty, \infty)$ (D) concave up on $(-\infty, 2)$; concave

down on $(2,\infty)$
(E) concave up on $(-\infty,-1)$; concave down on $(-1,0)$ and $(0,\infty)$

42. The inflection points of the function $f$ are described as follows:

(A) inflection point at $x=-1$ (B) no inflection points (C) inflection points at $x=-1$ and at $x=0$ (D) inflection points at $x=-1$, at $x=0$, and at $x=2$ (E) inflection points at $x=0$ and at $x=2$

43. Concerning the graph of $y=f(x)$, it is *NOT TRUE* that:

(A) there is a vertical tangent line at $(0,0)$ (B) the derivative does not exist at $x=0$
(C) the function $f$ takes on all real values (D) the derivative does not exist at $x=2$
(E) the function $f$ is defined for all real values of $x$.

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The next *TWO* questions concern a monopolist whose cost per unit of producing a product is \$3 and whose demand equation is

$$p=\frac{10}{\sqrt{q}}$$

44. The monopolist's profit is:

(A) minimized when $q=\frac{25}{9}$ (B) maximized when $q=\frac{25}{9}$ (C) maximized when $q=6$
(D) maximized when $q=\frac{9}{25}$ (E) minimized when $q=\frac{5}{3}$

45. In terms of price $p$, the monopolist's profit is:

(A) maximized when $p=\frac{1}{6}$ (B) minimized when $p=\frac{25}{9}$ (C) minimized when $p = \dfrac{9}{25}$

(D) minimized when $p = \dfrac{1}{6}$ (E) maximized when $p = 6$

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The next *THREE* questions concern the function

$$h(u,v) = \frac{8uv^2}{u^2+v^2}$$

46. The partial derivative $h_u(u,v) =$

(A) $\dfrac{8v^2}{2u}$ (B) $\dfrac{8}{v^2}$ (C) $\dfrac{(u^2+v^2)(8v^2)-(8uv^2)(2u)}{(u^2+v^2)^2}$ (D) $\dfrac{(u^2+v^2)(8u)-(8uv^2)(2u+2v)}{(u^2+v^2)^2}$
(E) $\dfrac{(8uv^2)(2u)-(u^2+v^2)(8v^2)}{(u^2+v^2)^2}$

47. The partial derivative $\dfrac{\partial}{\partial v}(h(u,v)) =$

(A) $\dfrac{(u^2+v^2)(16uv)-(8uv^2)(2v)}{(u^2+v^2)^2}$ (B) $\dfrac{16uv}{2v}$ (C) $\dfrac{8}{u^2}$ (D) $\dfrac{(8uv^2)(2v)-(u^2+v^2)(16uv)}{(u^2+v^2)^2}$
(E) $\dfrac{(u^2+v^2)(16v)-(8uv^2)(2v)}{(u^2+v^2)^2}$

48. If $\dfrac{\partial}{\partial v}(h(u,v)) = 0$ and $u \neq 0$ then:

(A) $v=1$ (B) $v=-1$ (C) $v=2$ (D) $v=0$ (E) $v=3$

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The last *TWO* questions concern products A and B, with corresponding prices $p_{\mathrm{A}}$ and $p_{\mathrm{B}}$ and corresponding demands, each depending on both prices, as follows:

$$q_{\mathrm{A}} = e^{-(p_{\mathrm{A}}+p_{\mathrm{B}})} \qquad \text{and} \qquad q_{\mathrm{B}} = \frac{16}{p_{\mathrm{A}}^2 p_{\mathrm{B}}^2}$$

49. If the price of A increases then demand for A:

(A) decreases only for low fixed prices of B
(B) decreases for any fixed price of B
(C) demand for A decreases for high fixed prices of B
(D) demand for A increases for low fixed prices of B
(E) demand for A increases for any fixed price of B

50. The products A and B can be classified as:

(A) competitive (B) neither competitive nor complementary (C) none of these
(D) inferior (E) complementary

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