

A2 Solutions

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a)
$$P\left(\frac{1}{X} \leq y\right) = P\left(X \geq \frac{1}{y}\right)$$

$$= \left(\frac{y\theta}{1+y\theta}\right)^\alpha$$

b)
$$P\left(X^{1/\tau} \geq y\right) = P\left(X \geq y^\tau\right)$$

$$= \left(\frac{\theta}{y^\tau + \theta}\right)^\alpha$$

c) With $\tau < 0$, $\tau \neq -1$ let $\tau^* = -\tau > 0$

$$P\left(X^{1/\tau} \geq y\right) = P\left(X^{-1/\tau^*} \geq y\right)$$

$$= P\left(\frac{1}{X^{\tau^*}} \geq y\right) = P\left(X \leq \frac{1}{y^{1/\tau^*}}\right)$$

$$= 1 - \left(\frac{\theta}{\theta + \frac{1}{y^{1/\tau^*}}}\right)^\alpha = 1 - \left(\frac{\theta}{\theta + y^{1/\tau}}\right)^\alpha$$

2.
a)

$$P(-\log X \leq y)$$

$$= P(X \geq e^{-y})$$

$$= e^{-e^{-y}} \quad -\infty < y < \infty$$

b) $P(-\sigma \log X + \mu \leq y)$

$$= P(X \geq e^{-\frac{y - \mu}{\sigma}})$$

$$= Q\left(\sqrt{2} \operatorname{erf}^{-1}(1 - e^{-e^{-\frac{y - \mu}{\sigma}}})\right)$$

$$-\infty < y < \infty$$

3.

$$P(x|\lambda) = \frac{\lambda^x e^{-\lambda}}{x!}$$

$$P(\lambda) = \Theta e^{-\lambda} \Theta$$

$$P(x, \lambda) = \Theta \frac{\lambda^x e^{-\lambda(\Theta+1)}}{x!}$$

$$P(x) = \int_{\lambda=0}^{\infty} \Theta \frac{\lambda^x e^{-\lambda(\Theta+1)}}{x!} d\lambda$$

$$= \frac{\int_0^{\infty} \frac{\lambda^{x+1} e^{-\lambda(\Theta+1)}}{x!} d\lambda}{(\Theta+1)^{x+1}}$$

$$= \binom{x+1}{x} \left(\frac{\Theta}{\Theta+1} \right) \left(\frac{1}{\Theta+1} \right)^x$$

$$x = 0, 1, \dots$$

4.

let $N = 0$ w.p. .6, $= 1$ w.p. .4

$$a) P(X=1) =$$

$$\begin{aligned} & P(X=1|N=0)P(N=0) \\ & + P(X=1|N=1)P(N=1) \\ & = e^{-1}(.6) + 2e^{-2}(.4) \end{aligned}$$

$$\begin{aligned} b) E(X|N=0) &= 1 \\ E(X|N=1) &= 2 \end{aligned}$$

$$\Rightarrow E(X) = 1(.6) + 2(.4) = 1.4$$

$$\begin{aligned} c) V(X|N=0) &= 1 \quad V(X|N=1) = 2 \\ \Rightarrow E V(X|N) &= .6(1) + .4(2) \end{aligned}$$

$$\begin{aligned} V(E(X|N)) &= .6(1-1.4)^2 + .4(2-1.4)^2 \\ &= .6(.16) + .4(.36) \\ &= 1.104 \end{aligned}$$

5.

$$p(x) = \frac{\lambda^x e^{-\lambda}}{x!}$$

$$= \frac{\frac{1}{x!} e^{x \log \lambda}}{e^{\lambda}}$$

$$p(x) = \frac{1}{x!}$$

$$r(\lambda) = \log \lambda$$

$$g(\lambda) = e^{\lambda}$$

b.

$$\lim_{\substack{x \rightarrow \infty \\ \theta \rightarrow \infty}} \left(\frac{\theta}{x + \theta} \right)^x$$

$$x \rightarrow \infty$$

$$\theta \rightarrow \infty$$

$$\frac{\theta}{x} = C$$

$$= \lim_{x \rightarrow \infty} \left(1 + \frac{x}{\theta} \right)^{-x}$$

$$= \lim_{x \rightarrow \infty} \left(1 + \frac{C x}{x} \right)^{-x}$$

$$= e^{-Cx} \quad , \quad x \gg \theta$$

3.

$$P(x=x)$$

$$= \int P(x=x) f_N(x) dx$$

$$4. P(x=1) = P(x=1 | N=0) P(N=0) + P(x=1 | N=1) P(N=1)$$

$$\begin{aligned} \text{N} & \leftarrow P(N=0) = 0.6 \quad P(N=1) = 0.4 \\ P(N=0) &= 0.6 \quad P(N=1) = 0.4 \end{aligned}$$



$$f_1(x) = 1, \quad 0 < x < 1$$

$$f_2(x) = 0.4 e^{-\lambda(x-1)}, \quad x > 1$$

$$f(x) = 0.6 f_1(x) + 0.4 f_2(x)$$

$$Q_1 + Q_2 = 1$$

$$Q_1 f_1(x) = Q_2 f_2(x)$$

$$Q_1 = Q_2 \lambda$$

$$Q_2 (1 + \lambda) = 1$$

$$Q_2 = \frac{1}{1 + \lambda}$$

$$Q_1 = 1 - \frac{1}{1 + \lambda} = \frac{\lambda}{1 + \lambda}$$

$$f_2(x) = \lambda$$

$$Q_2 f_2(x) = \frac{\lambda}{1 + \lambda}$$