Theorem 8.7:

$E(Y^L) = \alpha(1+r)\left[E(X \wedge \frac{u}{1+r}) - E(X \wedge \frac{d}{1+r})\right].$

Moments

If X and Y are random variables, and $E(Y|X)$ is conditional mean of Y given X, and $V(Y|X)$ is conditional variance of Y given X, then

$E(Y) = E(E(Y|X))$ and $V(Y) = E(V(Y|X)) + V(E(Y|X))$

$E(X \wedge d) = \int_{x=0}^{d} S_X(x)dx$

Risk Measures

- Value at Risk $VaR_p(X) = \pi_p$, is the 100 p'th percentile of the distribution of X.
- Tail Value at Risk
  $TVaR_p(X) = \frac{\int_{\pi_p}^{\infty} xf(x)dx}{1-p} = \frac{\int_p^1 VaR_u(X)du}{1-p} = \pi_p + \frac{\int_{\pi_p}^{\infty} S(x)dx}{1-p}$

Uniform

Density function	$f(x) = \frac{1}{b-a}$ (for $a < x < b$)
Survival funtion	$S(x) = \frac{b-x}{b-a}$ (for $a \leqslant x \leqslant b$)
Mean	$\frac{a+b}{2}$
Variance	$\frac{(b-a)^2}{12}$
Moment Generating Function	$M(t) = \frac{e^{bt}-e^{at}}{(b-a)t}$

Exponential

Density function	$f(x) = \frac{e^{-(\frac{x}{\theta})}}{\theta}$
Survival function	$e^{-\frac{x}{\theta}}$
Mean	θ
Raw moments	$\mu_n' = n!\theta^n$
Variance	θ^2
Moment Generating Function	$M(t) = \frac{1}{1-\theta t}$

$E[X \wedge d] = \theta(1 - e^{-d/\theta})$

Normal

Density function	$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$
Mean	μ
Variance	σ^2
Moment Generating Function	$M(t) = e^{\mu t + \frac{1}{2}\sigma^2 t^2}$

Binomial

Probability	$p_k = \binom{n}{k} p^k (1-p)^{n-k}$
mean	np
raw moments	$\mathbb{E}(X \cdots (X+1-m)) = n \cdots (n+1-m) p^m$
Variance	$np(1-p)$

Poisson

Probability	$p_k = e^{-\lambda} \frac{\lambda^k}{k!}$
mean	$\mu = \lambda$
raw moments	$\mathbb{E}(X(X-1) \cdots (X+1-m)) = \lambda^m$
Variance	λ
probability generating function	$P_X(z) = \mathbb{E}(z^X) = e^{\lambda(z-1)}$