ACSC/STAT 3703 - Winter 2026 - Assignment 2

Due: Friday, Feb 6, at 11:59 PM

1. Let X have a two parameter Pareto distribution with survivor function

$$S(x) = \left(\frac{\theta}{x+\theta}\right)^{\alpha}$$

where x, θ and α are all positive,

   (a) Derive the cdf or survivor function of the inverse distribution.

   (b) Derive the cdf or survivor function of the transformed distribution.

   (c) Derive the cdf or survivor function of the inverse transformed distribution.

2. Let X have an exponential distribution with survivor function $S(x) = e^{-x}$, $x > 0$.

   (a) Derive the distribution function of $-log(X)$, where log is the natural log.

   (b) Derive the distribution function of $-\sigma log(X) + \mu$, where $\sigma > 0$.

3. Let the conditional distribution of X given that $\Lambda = \lambda$ be a Poisson distribution with conditional probability mass function

$$p_{X|\Lambda)}(x|\lambda) = P(X = x|\Lambda = \lambda) = \frac{\lambda^x e^{-\lambda}}{x!},\ x = 0, 1, 2, \ldots$$

and suppose that Λ has an exponential distribution with survivor function $S(\lambda) = e^{-\lambda\theta},\ \lambda > 0$, where $\theta > 0$. Derive the unconditional distribution of X, also known as the marginal distribution.

4. X has a two component mixture distribution. With probability .6, X has a Poisson distribution with mean 1, and with probability .4, X has a Poisson distribution with mean 2.

   (a) Find the probability that $X = 1$.

   (b) Find $E[X]$. (Hint: imagine a random variable N which takes value 0 with probability .6, and takes value 1 with probability .4. Then calculate $E[X] = E[E[X|N]]$.)

   (c) Find $V(X)$. (Hint: use the conditional variance formula)

5. Show that the Poisson distribution with pmf

$$P(X = x) = \frac{\lambda^x e^{-\lambda}}{x!},\ x = 0, 1, 2, \ldots$$

is a member of the linear exponential family. That is, find the functions $p(x)$, $q(\lambda)$ and $r(\lambda)$.

6. For the Pareto distribution in problem 1, show that the limit of $S(x)$ as $\alpha \to \infty$ and $\theta \to \infty$ such that $\frac{\alpha}{\theta} = c$, a constant, gives the survivor function of the exponential distribution.