ACSC/STAT 3703 - Assignment 5, due Tuesday, March 17, 11:59 PM

1. Let X_1,X_2 and X_3 be independent random variables with probability mass function $P(X = 0) = 1/2$, $P(X = 1) = 1/4$, and $P(X = 3) = 1/4$.
   (a) Find the probability mass function of $S = X_1 + X_2$.
   (b) Find the probability mass function of $S = X_1 + X_2 + X_3$.

2. Suppose that X has an exponential distribution with mean 100, that there is an ordinary deductible of $d = 20$.
   (a) Out of 10 losses which are exponentially distributed with mean 100, how many would you expect to exceed the deductible of $d = 20$.? (Hint: first find $S_X(20)$. Then note that the number exceeding d is $\sum_{j=1}^{10} I[X_j > d]$ where $I[A]$ is the indicator of event A. Then use fact that mean of the sum is the sum of the means.)
   (b) In this situation (10 independent exponentially distributed losses with mean 100, a deductible of 20) what is the probability that exactly 9 of the losses exceed the deductible? (Hint: the number exceeding d has a binomial distribution. Which binomial distribution?)
   (c) Where $E(X \wedge d)/E(X)$ is the loss elimination ratio, what would be the deductible required to have a loss elimination ratio of 1/2?
   (d) Suppose now that in addition to a deductible d=20, there is a policy limit of $u = 200$, and a uniform inflation rate of 10%.
      i. What is the mean of the per loss variable Y_L?
      ii. What is the expected cost per payment Y_P?

3. Consider the aggregate loss in the individual risk model $S = \sum_{i=1}^{200} X_i$, where the base losses $X_1, X_2, \ldots$ are exponential with mean 100. Use a normal approximation to approximate $P(S > 22772)$. You can leave the answer as $P(Z > c)$, where c is to be determined.

4. Suppose that $X_1, X_2, \ldots, X_N$ are independent Poisson random variables with mean λ, and are independent of N which has a Poisson distribution with mean θ. Find the probability generating function of $S = \sum_{i=1}^{N} X_i$?

5. Using the recursion for the (a,b,0) class given in section 6.5, and the values of p_0, a and b given in table 6.1, find p_1, p_2 and p_3 for the binomial distribution with parameters $m = 6$ and $q = .25$.
   (a) find p_1^T, p_2^T, and p_3^T for the associated zero truncated random variable.
   (b) find p_1^M, p_2^M, and p_3^M for the associated zero modified random variable with $p_0^M = .5$.

6. Let X_1, X_2, ... be independent gamma random variables with parameters α and β, independent of N which has a Poisson distribution with mean λ.

   Let $S = \sum_{i=1}^{N} X_i$. (The X's might represent individual insurance claims, and N the number of claims in a year, in which case S is the total of claims in a year.) Find the mean and variance of S.