

Assignment 8 - due April 12, 11:59 PM

The exam has 9 questions. Show all of your work in the examination booklet.

Materials permitted: calculator.

- Let X_1, X_2, X_3 and X_4 be independent random variables with probability mass function $P(X = 0) = 3/4$ and $P(X = 1) = 1/4$. Find the probability mass function of $S = X_1 + X_2 + X_3 + X_4$.
- Suppose that X has a discrete distribution which is a finite mixture of a Poisson distribution with mean 2 and a Poisson distribution with mean 3, given by:

$$P(X = k) = .25 \frac{2^k e^{-2}}{k!} + .75 \frac{3^k e^{-3}}{k!}, \quad k = 0, 1, 2, \dots$$

- Calculate $P(X \geq 2)$.
 - Calculate the probability generating function of X , $P_X(z) = E(z^X)$.
 - Evaluate the mean of X as $P'_X(1)$.
 - Evaluate $E[X(X - 1)] = P''_X(1)$.
 - Combine the results of the previous two parts to obtain the variance of X .
- Suppose that N is a Poisson random variable with mean 100, and that losses X_1, X_2, \dots are i.i.d. having a normal distribution with mean 100 and standard deviation 20.
 - What is the mean of the aggregate claim in the compound model, for which $S = \sum_{i=1}^N X_i$?
 - What is the variance of $S = \sum_{i=1}^N X_i$?
 - What is $P(S > 200 | N = 2)$?
 - Do you think that the distribution of S will be well approximated by a normal distribution? Why or why not?
 - In the previous question, if the individual losses X_1, X_2, \dots instead had exponential distributions with common mean 100, what would be the mean of the aggregate claim $S = \sum_{i=1}^N X_i$?

5. Suppose that X has an exponential distribution with mean 100, that there is an ordinary deductible of $d = 20$.
- Out of 10 losses which are exponentially distributed with mean 100, how many would you expect to exceed the deductible of $d = 20$?
 - In this situation (10 independent exponentially distributed losses with mean 100, a deductible of 20) what is the probability that exactly 9 of the losses exceed the deductible?
 - Where $E(X \wedge d)/E(X)$ is the loss elimination ratio, what would be the deductible required to have a loss elimination ratio of $1/2$?
 - Suppose now that in addition to a deductible $d=20$, there is a policy limit of $u = 200$, and a uniform inflation rate of 10%.
 - What is the mean of the per loss variable Y_L ?
 - What is the expected cost per payment Y_P ?
6. Consider the aggregate loss in the individual risk model $S = \sum_{i=1}^{200} X_i$, where the base losses X_1, X_2, \dots are exponential with mean 100. Use a normal approximation to approximate $P(S > 22772)$.
7. Let X have an exponential distribution with mean 1.
- Find the value at risk at the 99% level, $Var_{.99}(X) = \pi_{.99}$.
 - Find the tail value at risk of X at the 99% security level, $TVaR_{.99}(X)$.
8. For holders of a certain nursing home insurance policy, the average length of a nursing home stay is 300 days, and 50% of the stays are terminated in the first 20 days. The terminations are uniformly distributed in that period. The policy pays 50 per day for the first 20 days, and 200 per day after that. What are the expected benefits payable for a single stay?
9. (a) Use probability generating functions to show that if X_1, X_2, \dots, X_n are independent Poisson random variables with mean λ , then $S = \sum_{i=1}^n X_i$ also has a Poisson distribution. What is the mean of that distribution?
- (b) Use probability generating functions to show that if X_1, X_2, \dots, X_N are independent Poisson random variables with mean λ , and are independent of N which has a Poisson distribution with mean θ , then $S = \sum_{i=1}^N X_i$ does NOT have a Poisson distribution.