Abstracts

Jonathan Borwein, Dalhousie University
Ramanujan's AGM continued fractions and their dynamics

An innnocent computational question—arising with an identity due to Ramanujan— led me, Richard Crandall, and our other collaborators into an exhilerating empirical ride through a rich garden of continued fractions, elliptic integrals, special functions and dynamical systems. I invite you to take a guided tour through this garden. It has now been tamed in that all is proven, as I shall indicate.

David Bradley, University of Maine
On q-analogs of multiple zeta-values and other multiple harmonic series

An example of a multiple harmonic sum is

$$Z_n(s,t,u) := \sum_{n \geq i \geq j \geq k} i^{-s} j^{-t} k^{-u}.$$

Here, the sum is over all positive integers i, j, k satisfying the indicated inequalities, which in some cases may be strict instead of weak as indicated. The bound n may be finite or infinite. The arguments s, t, u are unrestricted if n is finite, but are usually assumed to be positive integers, with $s > 1$ if n is infinite to ensure convergence. In general, we may have an arbitrary finite number of arguments, as opposed to only three.

The q-analog of a positive integer k is

$$[k]_q := \sum_{j=0}^{k-1} q^j = (1 - q^k)/(1 - q), \quad q \neq 1.$$

Naively, one can obtain a q-analog of multiple harmonic sums by replacing the summation indices by their respective q-analogs. However, it appears that this approach needs to be modified somewhat in order to yield interesting results. I plan to discuss what sorts of results can be obtained with appropriate modifications, outline a few of the techniques used to prove them, and hopefully give an indication of why researchers are interested in this subject.

Marc Chamberland, Grinnell College
An update on the 3x+1 problem

The $3x + 1$ Problem is a long-standing conjecture in number theory. Let T be a map from the positive integers into itself, where $T(x) = x/2$ if x is even and $T(x) = (3x+1)/2$ if x is odd. The conjecture asks whether, under iteration of the map T, any positive integer eventually reaches the value one. This talk gives a survey of the various approaches and results.

Robin Chapman, University of Exeter
Determinants of Legendre symbol matrices

We consider Hankel and Toeplitz matrices built from Legendre symbols. In patricular we consider their determinants. Computation suggests various conjectures concerning their evaluation; only some of which we have proved.

Stephen Choi, Simon Fraser University
Linear Diophantine Equation with Three Prime Variables

In this talk, we will discuss the small prime solutions of the following linear diophantine equation with three prime variables:

$$a_1p_1 + a_2p_2 + a_3p_3 = b.$$

Mark Coffey, Colorado School of Mines
New results on the Stieltjes constant: Proof of a Kreminski conjecture

The Stieltjes constants [1] ζ_k have been of interest for over a century, yet their detailed behavior remains under investigation. We have derived new relations amongst the values $\zeta_k(a)$ that are the expansion coefficients in the Laurent series for the Hurwitz zeta function. In this talk, we demonstrate the validity of one of the very recent conjectures of Kreminski [2] on the relationship between $\zeta_k(a)$ and $-\zeta_k(a + 1/2)$ as $k \to \infty$. In addition, we have developed a new integral relationship for the Hurwitz zeta function and one of its implications for the Stieltjes constants.

[1] T. J. Stieltjes, *Correspondance d'Hermite et de Stieltjes*, Volumes 1 and 2, Gauthier-Villars, Paris (1905).
[2] R. Kreminski, *Newton-Cotes integration for approximating Stieltjes (generalized Euler) constants*, Math. Comp. **72**, 1379-1397 (2003).

Eva Curry, Rutgers University and Dalhousie
Radix representations of $\mathbf{Z}^n$

An integer b gives a radix representation if there is a set of digits D such that every integer x can be written in the form $x = \sum_{j=0}^{N(x)} b^j d_j$, with the $d_j \in D$. This talk will present a generalization of this idea to $\mathbb{Z}^n$. I will give a sufficient condition for a matrix A to give a radix representation for $\mathbb{Z}^n$, and will discuss a generalization of signed radix representations. I will also show how the question of which matrices give a radix representation for $\mathbb{Z}^n$ is related to self-affine tilings of $\mathbb{R}^n$, and discuss connections to wavelets.

Karl Dilcher, Dalhousie University
A Pascal-type triangle characterizing twin primes

It is a well-known property of Pascal's triangle that the entries of the kth row, without the initial and final entries 1, are all divisible by k if and only if k is prime. In this talk I will present a triangular array similar to Pascal's that characterizes twin prime pairs in a similar fashion. The proof involves generating function techniques. Connections with orthogonal polynomials, in particular Chebyshev and ultraspherical polynomials, will also be discussed. This is joint work with K.B. Stolarsky.

Kevin Hare, University of Waterloo
Odd perfect numbers

A perfect number is a number n, such that the some of all perfect divisors of n sum to n. The first two examples are 6 and 28. All known examples of perfect numbers are even. The existance of odd perfect numbers is still undecided, and is the area of active research. This talk will discuss some of the history of the search for odd perfect numbers, as well as some of the computation techniques used to find lower bounds on the existance or non-existance of odd perfect numbers.

Jeff Hooper, Acadia University
TBA

Keith Johnson, Dalhousie University
Formal group laws and Legendre numbers

Lutz Lucht, Technische Universität Clausthal
Mean behaviour of arithmetic functions on certain thin sequences

Mean-value theorems with quantitative remainder term estimates are proved for large classes of arithmetic functions f on certain thin sequences $\mathfrak{a}$ of positive integers. These classes depend on the distribution of the divisors of $\mathfrak{a}$ and on convergence properties of the Dirichlet series of the quotient $\widetilde{f}(s)/\zeta(s)$, where $\widetilde{f}(s)$ is the Dirichlet series associated with f, and $\zeta(s)$ denotes the Riemann zeta function. The results are visualized for specific multiplicative functions f and sequences $\mathfrak{a}$.

Carl Pomerance, Dartmouth College
New primality criteria, after Agrawal, Kayal, and Saxena

Two years ago, M. Agrawal, N. Kayal, and N. Saxena announced their new deterministic, polynomial time primality test. They proved that their primality test would decide if n is prime or composite in about $(\log n)^{12}$ elementary steps, and they conjectured the true complexity was about $(\log n)^6$. Since then the exponent 12 has crept down due to their efforts and others, and it now stands at 6, the original conjecture, albeit for a somewhat different test. This last is a joint result of me and Hendrik Lenstra. Towards a more practical primality test, we even have a random version due to Bernstein, with expected running time of about $(\log n)^4$. Although the complexity estimates are sometimes not easy to establish, these tests are all based on succinct and essentially elementary primality criteria. These talks will give an overview of the different primality tests, and prove some of the criteria.

Kenneth B. Stolarsky, University of Illinois at Urbana-Champaign
Problems about polynomials related to Chebyshev's (discriminants, zeros, q-analogues, etc.)

Chebyshev polynomials are defined by a 3-term linear recurrence whose coefficients are (extremely simple) polynomials in x. We know their zero distribution because (e.g.) they are orthogonal polynomials. What happens if we move out of the orthogonality realm by changing the coefficient polynomials? Amazingly little seems to be known here. We indicate how in certain carefully chosen cases this may have connections with previous Dilcher-Stolarsky results on resultants of linear combinations of Chebyshev polynomials, and (more speculatively) how it may connect with the distribution of the powers $3^n \pmod{p}$.

Keith Taylor, Dalhousie University
Asymptotic properties of the spectra of Toeplitz matrices

Let T be the Toeplitz operator with symbol ϕ. Here ϕ is a bounded measurable function on the unit circle and T is an operator on l^2 that, considered as an infinite matrix, has the Fourier coefficient of ϕ at $i - j$ as its i, j entry. If the symbol is a polynomial, then T has only a finite number of nonzero diagonals (it is "band-limited"). For any positive integer n, let T_n denote the n by n upper left corner matrix of T. Szego theory gives the limiting distribution of the spectra of the T_n as n goes to infinity. We present an estimate on the rate at which the distribution of the spectra of the T_n approach this limiting distribution.

Alf van der Poorten, Centre for Number Theory Research, Sydney
Elliptic sequences and continued fractions

..., 2, 1, 1, 1, 1, 2, 3, 7, 23, 59, ... defined by $A_{h-2}A_{h+2} = A_{h-1}A_{h+1} + A_h^2$ and $A_0 = A_1 = A_2 = A_3 = 1$ arises from the curve $V^2 - V = U^3 + 3U^2 + 2U$ by reporting the denominators of the points $M + hS$, with $M = (-1, 1)$ and $S = (0, 0)$. The sequence $(\ldots, 2, 1, 1, 1, 1, 1, 1, 2, 3, 4, 8, 17, 50 \ldots)$ is given by the recursion $B_{h-3}B_{h+3} = B_{h-2}B_{h+2} + B_h^2$ and $B_0 = B_1 = B_2 = B_3 = B_4 = B_5 = 1$ and arises from adding multiples of the divisor at infinity on the Jacobian of the curve $Y^2 = (X^3 - 4X + 1)^2 + 4(X - 2)$ of genus 2 to the divisor given by $[(\varphi, 0), (\overline{\varphi}, 0)]$; here, no doubt to the joy of adherents to the cult of Fibonacci, φ is the golden ratio. Of course the real surprise is that the stated recursions produce sequences of integers.

Gary Walsh, University of Ottawa
On a diophantine equation of Cassels

J.W.S. Cassels gave a solution to the problem of determining all instances of the sum of three consecutive cubes being a square. This amounts to finding all integer solutions to the Diophantine equation $y^2 = 3x(x^2 + 2)$. We describe an alternative approach to solving not only this equation, but any equation of the type $y^2 = nx(x^2 + 2)$, with n a natural number. Moreover, we provide an explicit upper bound for the number of solutions of such Diophantine equations.

The method we present uses the ingenious work of Wilhelm Ljunggren, and a recent improvement by Florian Luca and the speaker. We further provide an upper bound for the number of solutions to the more general equation $y^p = nx(x^2+2)$, with p prime, by applying a very recent, and very deep, result of Michael Bennett. This is joint work with Florian Luca.