"The Birthday Paradox"

Example: One instructor and 72 students in the classroom. What is the probability that no student has the same birthday as the instructor? What is the probability that no students share a common birthday?

Answer: For the first question, $\frac{364^{72}}{365^{72}} = 0.82$. For the second question,

$$\frac{365 \times 364 \times 363 \ldots (365 - 72 + 1)}{365^{72}} < 1\%$$

This is called "The Birthday Paradox" because it seems counterintuitive that the probability would be so low in such a small number of people.

"The Birthday Paradox" (cont'd)

There is an *R* function `pbirthday` to evaluate the probability of a repeated birthday in a (randomly chosen) group of n people.

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> pbirthday(72)

[1] 0.9992134
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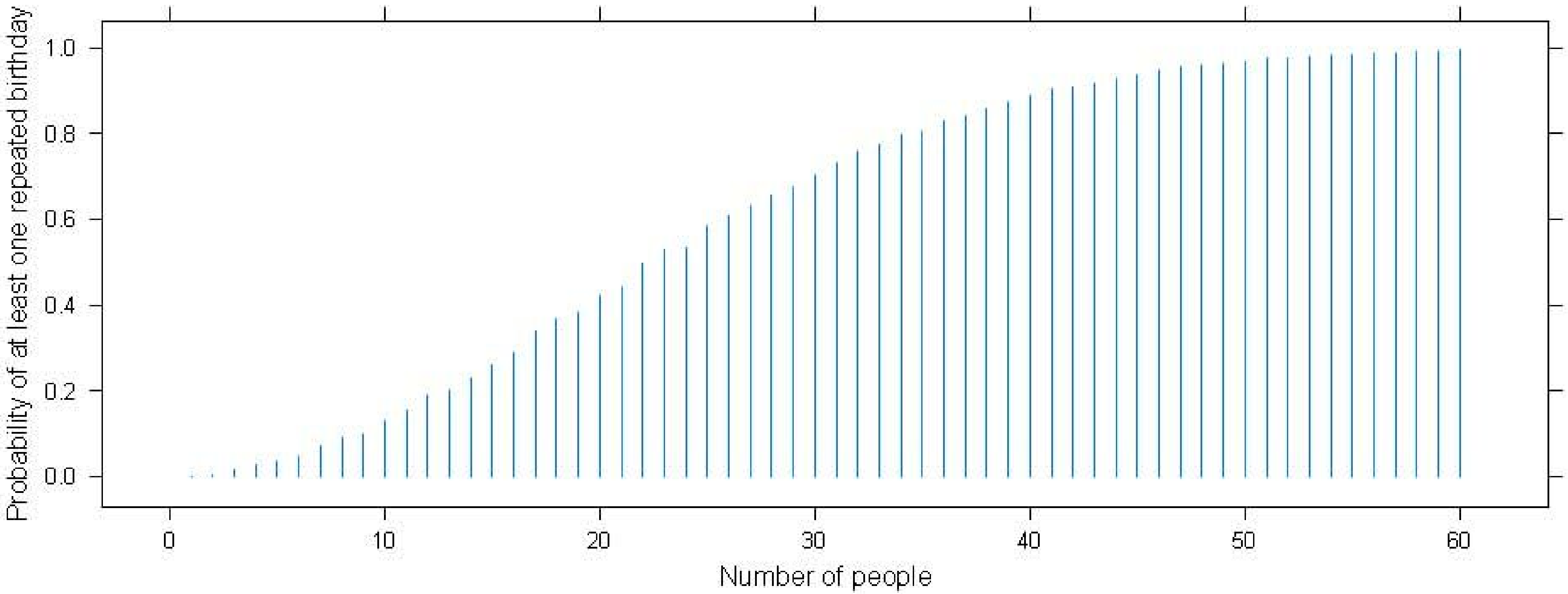

Examples

Example 1: If the probability that a research project will be well planned is 0.8 and the probability that is will be well planned and well executed is 0.72, what is the probability that a well planned research project will be well executed?

Answer: $0.72/0.8 = 0.9$.

Example 2: Roll a die twice. What is the probability that the total we get is 3? Given the information that the first number is 1, what is probability that the total we get is 3?

Answer: $2/36$ and $1/6$.

More on conditional probability

Sometimes the conditional probability can be determined easily, so we can actually use the conditional probability to calculate probability.

The multiplication rule: $P(A \cap B) = P(A|B)P(B) = P(B|A)P(A)$.

Example: There are 3 red balls and 2 blue balls in a box. Randomly take 2 balls from the box. What is the probability that both are red?

Answer: $P(R_1 \cap R_2) = P(R_1)P(R_2|R_1) = (3/5)(2/4) = 0.3$.

Another Example

Example: Three students have only one ticket to the super bowl. They will decide who is going in the following way: they will ask someone else to put the ticket in one of three boxes. One of them will pick and open one of the boxes. If the ticket is in the box, he gets it; otherwise, he is out, and the second person picks, and so on. Does it matter who picks first?

Answer: $P(\text{1st person get the ticket}) = 1/3$. The second person will get the ticket if 1st person not get the ticket, and the 2nd person pick the right one when facing the two remaining ticket. The probability is $(2/3)(1/2) = 1/3$.

The Tree Diagram

Example: In a factory three machines are used to produce computer parts. The defective rates for the three machines are 10%, 20%, and 25%. The first machine produces 60% of the products and the second machine produces 20%. What is the probability that a randomly chosen product is defective? What is the probability that a defective product is made by the first machine?

The Tree Diagram (cont'd)

Lie-detector tests are often routinely administered to employees in sensitive positions. Let $+$ $(-)$ denote the event that the lie-detector reading is positive (negative), Let T (L) denote the event that the subject is telling the truth (lie). According to studies of lie-detector reliability, $P(+|L) = .88$, $P(-|T) = .86$. Now suppose that lie-detector tests are routinely administered to screen employees for security reasons and that the vast majority have no reason to lie, so that $P(T) = .99$. A subject produces a positive response on the lie detector. What is the probability that he is in fact telling the truth?

Example

Example: Electrical engineers are considering two alternative systems for sending messages. A message consists of a word that is either a 0 or a 1. However, because of random noise in the channel, a 1 that is transmitted could be received as a 0 and vice versa. That is, there is a small probability, p, that

$$P(1 \longrightarrow 0) = p$$
$$P(0 \longrightarrow 1) = p$$

One scheme is to send a single digit. A second scheme is to repeat the selected digit three times in succession. At the receiving end, the majority rule will be used to decode. Compare the probabilities that a transmitted 1 will be received as a 1 under the two schemes.

Answer: for the second scheme, the probability is $(1 - p)^3 + 3p(1 - p)^2$.