Statistics 2060 - Midterm Exam
Date: Tuesday, February 27, 2007 Time: 10:05-11:25am

Name: _____ Student ID #: _____

This midterm exam has 3 pages with total of 30 marks. The number of points allocated to each portion of a problem is indicated on the left margin. The exam is closed notes and closed book, but one page of formula sheet (Legal sized paper with double sided writing) is permitted, and you may use a calculator. All your answers must be written on the exam papers. Partial marks can be given for partially correct answers.

1. Suppose that a random variable X has the probability mass function given in the following table:

x	0	1	2
$p(x)$	1/2	1/4	1/4

(2) (a) Calculate the mean of X, $\mathbf{E}(X)$.

$$EX = 0 \times \tfrac{1}{2} + 1 \times \tfrac{1}{4} + 2 \times \tfrac{1}{4} = \tfrac{3}{4} \text{ or } 0.75$$

(2) (b) Calculate the variance of X, $\mathbf{V}(X)$.

$$V(X) = EX^2 - (EX)^2$$
$$= 0 \times \tfrac{1}{2} + 1 \times \tfrac{1}{4} + 4 \times \tfrac{1}{4} - \left(\tfrac{3}{4}\right)^2 = \tfrac{11}{16} \text{ or } 0.6875$$

(2) (c) Calculate $\mathbf{E}[e^X]$.

$$E[e^X] = e^0 \times \tfrac{1}{2} + e^1 \times \tfrac{1}{4} + e^2 \times \tfrac{1}{4} = \tfrac{1}{2} + \tfrac{e}{4} + \tfrac{e^2}{4}$$
$$= 3.0268$$

2. Suppose event A has $P(A) = 0.2$, and $P(B|A') = 0.5$. Find

(2) (a) P(B) if A and B are independent.

$$P(A') = 0.8$$
$$P(B|A') = \frac{P(A' \cap B)}{P(A')} = 0.5 \qquad P(A' \cap B) = 0.4$$
$$P(B) = \frac{0.4}{P(A')} = \frac{0.4}{0.8} = 0.5$$

(2) (b) P(B) if A and B are mutually exclusive.

$$P(B|A') = 0.5 \qquad P(A' \cap B) = 0.4$$

$$B \subset A'$$
$$A' \cap B = B$$

$$P(B) = 0.4$$